Capacity Reservation for Time-Sensitive Service Providers: An Application in Seaport Management

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Abstract

This paper studies a capacity management problem in which a facility provider offers its facility to two service providers. The facility provider can either pool the service providers together to share the facility or reserve a dedicated facility for the service providers. The service providers determine their service capacity levels to serve each market with linear time-sensitive demand. We assume that both the facility provider and the service providers maximize demand rates. We find that the facility provider’s optimal capacity strategy critically depends on the ratio of the service providers’ demand loss rates, which are the demand loss per unit time increase. The facility provider prefers the reservation strategy to the pooling strategy if one service provider’s demand loss rate is significantly larger than the other’s. Otherwise, the facility provider prefers the pooling strategy. From the service providers’ perspective, the service provider who has a large demand loss rate prefers the reservation strategy, but the service provider who has a small demand loss rate prefers the pooling strategy. We also study a centralized system, in which the facility provider fully controls the service providers and hence is able to eliminate the negative effect of the facility competition between the service providers. We find that pooling is always optimal for the facility provider, which suggests that facility capacity competition is a prerequisite condition for not pooling the service providers together. We connect these managerial insights with the operations of seaports in Eastern Asian area (e.g., Hong Kong, Shanghai and etc).

Keywords: capacity game, capacity management, port management, time-based competition.
1 Introduction

The world’s economies are becoming more interrelated as a result of increasing international trading and globalization. Worldwide merchandise exports have been growing at an average rate of 5.5% per year since 2000 and the total exports and imports across the world were over $27.5 trillion in 2007 (The World Trade Organization 2008). China has emerged as a manufacturing engine for the globe. Its exports were $1.43 trillion (approximately 32% of GDP) and imports were $1.13 trillion (approximately 26% of GDP) in 2008 (National Bureau of Statistics of China 2009).

Seaports play an increasingly important role to connect the world and support the globalization trend. The world’s port throughput has had an annual growth rate of 10% since 2000 (The World Bank 2007, Module 2, Page 49) and the largest ten container ports in the world, among which six of them are located in China (Shanghai, Hong Kong, Shenzhen, Guangzhou, Ningbo-Zhoushan and Qingdao), handled 172 million TEUs (twenty-foot equivalent units) in 2008 (Hong Kong Marine Department 2009a). Seaports provide facilities for carriers to dock their vessels and load/unload their cargos, and they have significant impacts on the operational efficiency of the carriers. The port service industry reported about $50-55 billion in business receipts in 2006 and this amount is expected to grow to $75-80 billion in 2009 (The World Bank 2007, Module 1, Page 7).

The total capital investment required to build a modern container port is often enormous. For instance, the Port of Tanjung Pelepas in Malaysia, which started operating in 1999, required a capital investment of $745 million (The World Bank 2007, Module 2, Page 28). Capacity expansion of an existing port is also expensive. The estimated cost to redevelop the existing Rajiv Gandhi Container Terminal in India and build a new terminal nearby was about $460 million in 2004 (The World Bank 2007, Module 3, Page 73). To recover the enormous capital investment, a port has to handle a large volume of vessels and cargos to achieve economies of scale. For port owners and operators, an important objective is to keep port facilities busy and to use them efficiently.

But a busy port may cause long and unpredictable time delays for carriers. For instance, the average containership waiting time at the Port of Cartagena in Colombia is about two hours and when the berth occupancy rate (a measure of port utilization) is 50%, but the time jumps to 10 days when the berth occupancy rate increases to 90% (The World Bank 2007, Module 1, Page 2). Long time delay in a port often causes a loss to carriers as their customers are usually sensitive to the time spent on the transportation route. To reduce the time delay and avoid competing with other carriers for port facilities, carriers are inclined to having dedicated port facilities (i.e., facilities that can only be used by the designated carrier). By using dedicated facilities, carriers gain more control over port operations and ensure better service quality than by sharing facilities with other carriers.

A strategic problem for a port authority is to decide whether to pool all carriers together to
share the port facilities or to allocate the facilities to individual carriers. When a port pools the vessels from all carriers together and fully utilizes its facilities, this pooling effect generally leads to more efficient use of the facilities. In contrast, the port partially loses the pooling effect if it reserves part of its facilities for a specific carrier, since dedicated facilities cannot be used to handle other carriers’ vessels even when they are idle. However, the pooling strategy is not perfect for the port. When carriers are put together, they may compete for the port facilities by increasing the vessel frequency in order to provide better service for their customers. This competition effect may result in congestion and offset the benefit of the pooling effect. The competition effect exists in logistic systems in which service users share public facilities. For instance, the traffic jam at major U.S. airports in the summer of 2007 was partially caused by the increase in flight frequencies of U.S. airline companies, who intended to offer convenient schedules for travelers (Business Week 2007). Since using dedicated facilities separates the operations of different carriers, this reservation strategy eliminates the competition effect as well as the pooling effect. Hence, it is important for both a port authority and carriers to understand the tradeoffs between pooling and reservation strategies and the interactions among their capacity decisions.

In this paper, we consider a facility provider (e.g., a destination seaport) that offers its facility to two service providers (e.g., carriers), who ship cargos from two different origin ports to the same destination port. The demand rate at each origin port is measured by the number of vessels required to ship cargos out per unit time. Each service provider decides its service capacity (the shipping frequency of vessels on its route), which is also the demand rate of the service provider at the destination port. The facility’s capacity is measured as the number of vessels capably handled per unit time. The facility provider chooses either to let the service providers share the facility (by the pooling strategy) or to allocate a dedicated facility to the service providers and serve them separately (by the reservation strategy).

The demand rate of the service and the shipping frequency determine the time that cargo spends at the origin port. Similarly, the shipping frequency and the facility capacity determine the time that vessels spend at the destination port of the route. We assume that the demand rate of the service decreases linearly in the total processing time spent at the origin and destination ports. We also assume that both the facility provider and the service providers maximize their demand rates. Both assumptions are consistent with the practice in the maritime industry. It has long been known that shipping in a timely fashion is an important business aim in transportation industries. Due to large capital investment, achieving economies of scale and high capacity utilization is critical for the success of both ports and carriers. This makes cargo volume one of the most important performance measures in the industry (see, e.g., Stopford 1997 and The World Bank 2007, Module 3, Page 85).

The focus of our study is on identifying conditions under which the facility provider and service
providers should adopt the pooling/reservation strategy and optimize their capacity decisions. We consider three scenarios. In the first scenario, the facility provider adopts the pooling strategy. The service providers determine their service capacity levels and compete for facility usage. We find a unique Nash equilibrium for this scenario. In the second scenario, the facility provider allocates facility capacity to each service provider, who determines the service capacity given its dedicated facility capacity. Finally, in the third scenario, we study the first-best outcome of a centralized system, in which a central planner jointly chooses the facility capacity management strategy and the service capacity levels. We obtain closed-form solutions for all three scenarios.

We find that the facility provider’s optimal choice between the pooling strategy and the reservation strategy critically depends on the ratio of the demand loss rates of the service providers. The demand loss rate is defined as the demand loss if the total processing time increases by one unit. It is proportional to the service provider’s potential market size (i.e., the potential volume of cargos on a route) and the time sensitivity of demand. The facility provider prefers the reservation strategy if one service provider’s demand loss rate is much larger than the other’s (e.g., the demand loss ratio is greater than 14 in Theorem 5); otherwise, it prefers the pooling strategy. Our results are consistent with industry practice. For instance, the Maersk Line accounts for almost 80%-90% of the traffic at the Port of Salalah and dominates the rest of small carriers at the port. Hence, it is not surprising that the port provides Maersk with dedicated facilities. However, at the ports of Singapore, Hong Kong and Shanghai, which are the three largest ports in the world, none of the carriers has such a clear size dominance over other carriers. Hence, these ports do not provide dedicated facilities (The World Bank 2007, Module 3, Page 86).

We also find that if one service provider’s demand loss rate is four times larger than the other’s, the dominant service provider prefers the reservation strategy (Theorem 7). This is why large carriers often request dedicated facilities. But their requests may be denied by a port authority, who only grants dedicated facilities if the demand loss ratio is larger than 14 (Theorem 5). In contrast, for a service provider, whose demand loss rate is less than the other’s, the pooling strategy is preferred.

Finally, we study a centralized system, in which the facility provider fully controls the service providers and hence is able to eliminate the negative effect of the facility competition between the service providers. We find that pooling is always optimal for the facility provider, which suggests that facility capacity competition is a prerequisite condition for not pooling the service providers.

The rest of the paper is organized as follows. In Section 2, we provide a brief review of the related topics in the literature. The model is introduced in Section 3. Then, we study the pooling strategy, the reservation strategy and the centralized system in Sections 4, 5 and 6, respectively. We make comparisons between the pooling and reservation strategies in Section 7 and draw conclusions in Section 8. All proofs are included in the Appendix.
2 Literature Review

Speed has become an important strategic tool for firms to gain market share (Stalk and Hout 1990 and Blackburn 1991). For service and logistic companies, fast delivery is a key to attracting customers.

Service speed has been widely studied in the operations management literature. Kalai et al. (1992) studied a duopoly game in which two firms compete for market shares by choosing individual server capacity levels in a queueing system. Li (1992) studied an oligopoly game in which firms stock up inventories to cut delivery time and attract time-sensitive customers. Li and Lee (1994) studied price competition in a time-sensitive market and found that a firm with high processing capacity enjoys a price premium. Lederer and Li (1997) developed a competition model in which customers have different sensitivity to delivery time and firms can differentiate themselves by pricing, production and scheduling policies. So (2000) studied a price and delivery time competitive game in which firms satisfy customer demand within a guaranteed delivery period at a prefixed probability level. He found that high capacity firms offer better time guarantees than do low capacity firms and an increase of time sensitivity in customer demand strengthens this differentiation. Boyaci and Ray (2003) studied a product differentiation problem in which a firm determines the prices of a regular and an express product and the delivery time of the express product. They examined the relationship among capacity cost, time differentiation and price differentiation. Allon and Federgruen (2008) studied price and waiting time competition with general queueing systems and examined how the competitive behavior of service providers changes with the queueing system characteristics. Chayet and Hopp (2007) studied a sequential entry game in which firms compete on capacity, price and lead time. They found that the entrant needs superior operational capability to overcome the incumbent’s first-mover advantage.

The existing delivery time literature focuses on a simple service system, in which a service provider can deliver the service by herself and hence is always able to shorten the delivery time and improve service quality by increasing her service capacity. However, in practice, many service systems involve with multiple parties and increasing the service capacity of one service provider may not help improving the overall service system performance. For instance, a maritime system includes carriers and port authorities. Since a seaport has a limited capacity in processing vessels, a carrier may not be able to shorten the cargo delivery time when pushing the vessel frequency close to the port’s handling capacity. Similar phenomena occur in many logistic systems, in which service providers share logistic assets (e.g., ports, roads and other public facilities) to provide service to customers and gain economies of scales (Fuller et al. 1993). However, when traffic is heavy in these systems, public logistic facilities become congested, which causes delays and hurts service quality. Unlike the previous literature, this paper studies the impacts of both service and public facility
capacity levels on the delivery time.

Moreover, the bottleneck of public logistic facilities becomes severe when multiple service providers compete on a fixed amount of facility capacity. It is well known that a user of a public resource often ignores the negative externality that she/he imposes on other users (e.g., Haviv and Ritov 1998). This ignorance is the cause of the congestion in many logistic systems (as in the airport congestion case mentioned in Section 1). One way to solve this issue is to use incentive-compatible pricing schemes (see, e.g., Mendelson and Whang 1990 and Ha 1998), which are widely adopted by public transportation authorities. Another way is to allocate dedicated facilities to certain types of users, which is commonly practiced by port authorities and is the focus of this paper.

It is well known that combining separate subsystems into one system may improve the overall system efficiency, since the combination reduces the chance of idleness of subsystems and generates economies of scale (see, e.g., Smith and Whitt 1981 and Whitt 1999). However, if customers have heterogenous characteristics, then merging queues may be counterproductive. For instance, if customers fall into classes with different service time distributions, then keeping different types of customers into separate queues may be optimal (Smith and Whitt 1981 and Whitt 1999). Yu et al. (2009) studied a capacity management problem, in which a set of independent firms can share capacity. They found that capacity pooling may not be optimal if the workloads of firms are significantly different. Another reason of not merging queues is that customers have different time sensitivity parameters. Pangburn and Stavrulaki (2008) studied a joint pricing and capacity management problem and found that capacity pooling is suboptimal if customers are heterogenous in their time sensitivity. See other reasons of not merging queues in Rothkopf and Rech (1987).

In our paper, the pooling decision for the facility provider not only depends on the characteristics of the service providers (i.e., the demand loss rate) but also whether or not there exists competition for facility usage. In the decentralized case (i.e., the facility provider does not control the service providers), if the characteristics of the service providers are significantly different, then the pooling strategy is not optimal for the facility provider. This is because competing for facility usage between the service providers creates congestion and delays, which offset the pooling benefit. In contrast, in the centralized system (i.e., the facility provider fully controls the service providers), the pooling strategy is always optimal for the facility provider, since there is no facility capacity competition. Hence, one of the important reasons of not pooling self-interested service providers is the negative effect of facility capacity competition.

3 The Model

In this section, we first introduce a general framework for the port system in our problem. Second, we propose a stylized model that is analytically tractable. Finally, we summarize a three-month investigation at Kwai Tsing Container Terminals of Hong Kong, which provides some empirical
observations to support the stylized model.

3.1 The Problem Framework

We consider a facility provider that provides facilities to service providers. For instance, a port provides berths to carriers, who ship or transship cargos to and from the port. The facility provider has a total facility capacity of $K$, which is measured as the number of vessels that the facility can handle per unit time. Throughout the paper, we assume that the total facility capacity is fixed, since it is often very time-consuming and/or expensive to change the facility capacity.

For simplicity, we consider only two service providers. This is sufficient to demonstrate the tradeoff between the capacity reservation and pooling strategies. We assume that the service providers serve different markets that do not interact with each other. For instance, carriers may serve different routes, which have different origins but share the same destination port (see Figure 1). In this case, the demands for the carrier’s services rarely affect each other. We let $\lambda_i$ be the demand rate for service provider $i$, which is measured as the number of vessels needed to ship the cargo out of the origin port per unit time, and assume that the demand rate strictly decreases in the total delivery time of service $i$. The service providers determine their service capacity levels (e.g., the frequency of vessels traveling on the route). We denote the service capacity of service provider $i$ as $\mu_i$, where $i = 1, 2$.

In the maritime industry, the total transportation time of cargos can be decomposed into three phases: the dwell time $t_d$ for the next available vessel at the origin port, the shipping time on the ocean, and the facility time $t_f$ that a vessel spends waiting for and using port facilities at the destination port. Since the second phase of the total transportation time is independent of the service and facility capacity levels, we let the total processing time $t_i = t_d + t_f$ and assume that the demand rate $\lambda_i = D_i(t_i)$ is decreasing in $t_i$.

In the dwell phase, only when both the facility and vessels of service providers are available at the origin port, cargos can be loaded and shipped out. To focus on the interaction between the
destination port and carriers’ capacity decisions, we assume that the facility at the origin port is capable of handling cargos and always available whenever there is a vessel arrived. This is likely to occur if the origin port has a dedicated facility for service providers. Hence, the dwell time for service provider \( i \) is a function of the demand rate \( \lambda_i \) and service capacity \( \mu_i \), which is denoted as \( t_d(\lambda_i, \mu_i) \). We assume that \( t_d(\lambda_i, \mu_i) \) is increasing in the demand rate \( \lambda_i \), but decreasing in service capacity \( \mu_i \). The dwell time \( t_d(\lambda_i, \mu_i) \) should go to infinity as \( \lambda_i \) and \( \mu_i \) get close to each other (i.e., there is no sufficient service capacity to handle the demand).

In the facility phase, the facility time is a function of the facility capacity and the total amount of vessel traffic that uses the facility. If service providers share the facility, the facility time function is \( t_f(\mu_i + \mu_{-i}, K) \). If service provider \( i \) has dedicated facility capacity \( K_i \), the facility time is \( t_f(\mu_i, K_i) \). The facility time function is expected to decrease in the facility capacity level, but increase in the amount of vessel traffic. The facility time should go to infinity as the amount of vessel traffic approaches to the facility capacity level.

As a justification of these properties, we consider the example of the Port of Cartagena in Colombia (The World Bank 2007, Module 1, Page 2). In 1993, the average berth occupancy rate (the utilization of the facility capacity) was 90% and the containership waiting and turnaround time (the facility time) was about 13 days. The port increased its capacity significantly since then. In 2003, the average berth occupancy rate was 50% and the containership waiting and turnaround time was reduced to less than 9 hours. During the same period, due to the significant increase in vessel frequency (service capacity), the cargo dwell time was also reduced from more than 30 days to only 2 days for cargo shipped to other ports.

By the monotonic properties of the dwell and facility times, the total processing time \( t_i \) is expected to decrease in the facility capacity level, but increase in the demand rate. The total processing time \( t_i \) should go to infinity as the service capacity level (the frequency of vessels) approaches to either the demand rate or the facility capacity level. Hence, the total processing time should be a U-shaped curve of the service capacity level. These generic properties should hold in maritime systems.

Notice that the total processing time and demand rate interact with each other. When service providers share the facility, the equilibrium demand rate \( \lambda_i \) is determined by

\[
\lambda_i = D_i(\lambda_i(\mu_i) + t_f(\mu_i + \mu_{-i}, K)).
\]  

Since \( D_i(t_i) \) is decreasing in \( t_i \) and \( t_d(\lambda_i, \mu_i) \) is increasing in \( \lambda_i \), Equation (1) determines a unique demand rate function \( \lambda_i(\mu_i, \mu_{-i}, K) \) given \( \mu_i, \mu_{-i} \) and \( K \). Similarly, if service provider \( i \) has a dedicated facility \( K_i \) at the destination port, the demand rate function \( \lambda_i(\mu_i, K_i) \) is the solution of \( \lambda_i = D_i(\lambda_i(\mu_i) + t_f(\mu_i, K_i)). \)

Due to the large capital investment needed to build ports and purchase containerships, the total
cargo volume is a primary concern of both ports and carriers. For instance, the World Bank pointed out that the major concern of carriers is their market share while their port handling charges are of secondary importance (The World Bank 2007, Module 3, Page 85). To a port authority, the overall contribution of the port operations to the local economy, which is correlated with the cargo volume, is often more important than its own profitability. Hence, we assume that the facility provider and the service providers maximize their demand rates.

When the pooling strategy is adopted, each service provider’s capacity decision affects the other’s total processing time and hence demand rate. We can model the strategic interaction between service providers’ capacity decisions as a simultaneous game and look for the Nash equilibrium of the game. When the reservation strategy is adopted by the facility provider, there is no strategic interaction between service providers’ capacity decisions. The facility provider decides how to allocate its total capacity $K$ to two service providers with the objective of maximizing the total demand rate. The preference over the pooling and reservation strategies can be determined by comparing all parties’ performance under the two facility management strategies.

The problem framework we described is generic. By plugging in the exact form of the total processing time and demand functions, the tradeoffs between pooling and reservation strategies can be made at least via a simulation-based approach. The exact form of the total processing time is determined by the operational details of the maritime system (as shown in Figure 1), such as, the operational policies of the origin and destination ports and carriers. Given the complexity of the maritime system, it is extremely cumbersome, if possible, to derive a closed-form of the total processing time. Since the pooling decision is strategic for the facility provider and has long-term impact on all parties in the maritime system, we avoid to specify the detailed operational information of the maritime system, which tends to change over time. Instead, we choose to use a stylized model of the total processing time, which is mathematically tractable and enables us to gain managerial insights. These managerial insights can be formed as rules of thumb for strategic seaport capacity management.

### 3.2 The Stylized Model

We assume that the total processing time has a simple formula, that is, $t_i = \frac{1}{K_i - \mu_i} + \frac{1}{\mu_i - \lambda_i}$, where $K_i$ is the facility capacity available to service provider $i$. Notice that $K_i$ represents the dedicated facility capacity for service $i$ if the facility provider adopts the reservation strategy, or $K_i = K - \mu_i$ if the facility provider adopts the pooling strategy. The above formula satisfies all generic properties of the time functions introduced in Section 3.1 and hence captures the strategic impact of the facility and service capacity levels on the delivery time. Moreover, we provide some empirical evidence to support this specific form of the total processing time function in Section 3.3.

We assume a linear time-dependent demand function, that is, $D(t_i) = A_i(1 - \theta_i t_i)$, where $A_i$
is the potential market size and $\theta_i$ is the time sensitivity parameter. Notice that we normalize the shipping time on the ocean as zero (this can be done by adjusting the parameters of the demand function). The linear demand form is widely adopted in the literature (see, e.g., Kalai et al. 1992 and So 2000).

With the exact form of the total processing time and demand functions, we have the demand rate, as a function of $K_i$ and $\mu_i$,

$$\lambda_i(K_i, \mu_i) = \frac{1}{2} \left[ A_i - \frac{A_i \theta_i}{K_i - \mu_i} + \mu_i - \sqrt{\delta_i^2 + 4A_i \theta_i} \right],$$

(2)

where $\delta_i = A_i - \frac{A_i \theta_i}{K_i - \mu_i} - \mu_i$, if $\frac{1}{2} \left( K_i - \sqrt{K_i^2 - 4K_i \theta_i} \right) < \mu_i < \frac{1}{2} \left( K_i + \sqrt{K_i^2 - 4K_i \theta_i} \right)$ and $K_i > 4\theta_i$; otherwise, $\lambda_i(K_i, \mu_i) = 0$. (See derivation details in the Appendix.)

We solve the service provider’s demand maximization problem as shown in Proposition 1.

**Proposition 1** Assume that $K_i > 4\theta_i$. (1). The optimal service capacity is $\mu_i^*(K_i) = \frac{3K_i + A_i}{4} - \frac{1}{2} \sqrt{(K_i - A_i)^2 + 16A_i \theta_i}$; (2). The optimal demand rate is $\lambda_i^*(K_i) = \lambda_i(K_i, \mu_i^*(K_i)) = \frac{K_i + A_i}{2} - \frac{1}{2} \sqrt{(K_i - A_i)^2 + 16A_i \theta_i}$; (3). $\mu_i^*(K_i) > \lambda_i^*(K_i) > 0$; (4). The optimal service capacity $\mu_i^*(K_i)$ and the demand rate $\lambda_i^*(K_i)$ are increasing in $K_i$ and $A_i$, but decreasing in $\theta_i$.

By Proposition 1, as the available facility capacity $K_i$ and/or the market size $A_i$ increase, service provider $i$ increases its service capacity to attract customer demand. But as customers become more time sensitive (i.e., an increase of $\theta_i$), the demand for the service drops, which reduces the required service capacity. The condition of $K_i > 4\theta_i$ in Proposition 1 implies that customers are not extremely time sensitive and/or the available facility capacity is moderately large. An invalidation of this condition shuts down the operations of service provider $i$ (i.e., $\mu_i^*(K_i) = \lambda_i^*(K_i) = 0$). Since this is a trivial case, we will avoid it in the rest of this paper.

### 3.3 Empirical Observations

We conducted a three-month investigation (from October 9, 2008 to January 14, 2009) on the vessel arrival and processing pattern at Kwai Tsing Container Terminals of Hong Kong, which has been the third busiest container port in the world since 2007, just after Singapore and Shanghai. The data source is from the Vessel Traffic Management System Reports published by Hong Kong Marine Department (Hong Kong Marine Department 2009b). These reports enable us to trace key time points of a vessel in and out of Kwai Tsing Container Terminals. In these reports, there are four key time variables: ETA (Estimated Time of Arrival), ATA (Actual Time of Arrival), ETD (Estimated Time of Departure) and ATD (Actual Time of Departure). There are 636 vessel records with complete information about the four key time variables during the investigation period.

ETA was announced approximately three days before the arrival of vessels. We let $DTA = ATA - ETA$ be the time difference between the actual arrival time and estimated arrival time.
Among the 636 vessel records, 4 vessels arrived at least 30 hours before the estimated arrival time (i.e., $DTA < -30$) and 24 vessels arrived at least 30 hours after the estimated arrival time (i.e., $DTA > 30$). The average value of $DTA$ is 5.54 hours and its standard deviation is 8.69 hours. This implies that vessel arrivals are affected by random factors and are not perfectly scheduled. Hence, vessel arrivals should be modeled via a stochastic process (e.g., a Poisson process). Similarly, our data implies that the vessel departure time is neither perfectly scheduled (i.e., $ETD$ and $ATD$ are different in most vessel records). Hence, the time that a vessel spends at the port is uncertain.

We let $PT = ATD - ATA$, which is the time period that a vessel spends at the port. The mean and standard deviation of $PT$ are 12.03 hours and 5.12 hours respectively. Figure 2 shows the histogram of $PT$. This histogram is right-skewed and indicates that the distribution of $PT$ may be an Erlang distribution. In the stylized model, the total processing time is composed of the formula of the expected system time of an M/M/1 system. Notice that an exponential distribution is an Erlang-1 distribution. Hence, our data roughly supports the total processing time function in the stylized model. In Sections 4-7, we show that this simple formula of the total processing time function results in clean managerial insights, which can be used as rules of thumb for strategic seaport capacity management.

4 The Pooling Strategy

When the facility provider adopts the pooling strategy, the service providers determine their individual service capacity levels and compete on the facility capacity. The available facility capacity to service provider $i$ is $K_i = K - \mu_{-i}$, which depends on the service capacity of the other service provider. This implies that each service provider’s capacity decision affects the other’s capacity decision. We model this as a simultaneous game and study the pure strategy Nash equilibrium of the game.
We let $\mu_i^{POOL^*}$ and $\lambda_i^{POOL^*}$ respectively denote the equilibrium service capacity level and demand rate of service provider $i$, and we let $A^{POOL^*} = \lambda_1^{POOL^*} + \lambda_2^{POOL^*}$ denote the total demand rate of the facility provider under the equilibrium. To ensure that both service providers use the facility, we make the following assumption.

**Assumption 1** $K \geq \max(A_1, A_2) + 8 \max(\theta_1, \theta_2)$.

This assumption implies that the total facility capacity is larger than the potential market size of each service provider. We let $\mu^*_i(K_i)$ be the best response function of service provider $i$. By Claim 1 of Proposition 1 and Assumption 1,

$$K - \mu^*_i(K_i) = \frac{K - A_{i-1}}{4} + \frac{1}{4} \sqrt{(K - A_{i-1})^2 + 16A_{i-1}\theta_{i-1}} > (K - A_{i-1})/2 > 4\theta_i$$

for $i = 1, 2$. By Claim 4 of Proposition 1, $\mu^*_i(K_i) \leq \mu^*_i(K)$ for any $K_i \leq K$, where $K_i = K - \mu_{i-1}$. Hence, $K - \mu^*_i(K_{i-1}) > 4\theta_i$ for any $K_{i-1} \leq K$ and the condition of Proposition 1 always holds at any equilibrium. By Claim 3 of Proposition 1, $\mu^*_i(K_i) > \lambda^*_i > 0$ for $i = 1, 2$. Hence, Assumption 1 implies that both service providers use the facility under the pooling strategy.

By Claims 1 and 2 of Proposition 1, the equilibrium service capacity $\mu_i^{POOL^*}$ and the demand rate $\lambda_i^{POOL^*}$ must satisfy the following equations:

$$\mu_i^{POOL^*} = \frac{3(K - \mu^{POOL^*}_{i-1}) + A_i}{4} - \frac{1}{4} \sqrt{(K - \mu^{POOL^*}_{i-1} - A_i)^2 + 16A_i\theta_i},$$

$$\lambda_i^{POOL^*} = \frac{(K - \mu^{POOL^*}_{i-1}) + A_i}{2} - \frac{1}{2} \sqrt{(K - \mu^{POOL^*}_{i-1} - A_i)^2 + 16A_i\theta_i},$$

for both $i = 1, 2$. By solving the equations through various transformations, we obtain a unique pure strategy Nash equilibrium in Theorem 1.

**Theorem 1** If Assumption 1 holds, then there exists a unique pure strategy Nash equilibrium such that both service providers use the facility. At the equilibrium, the service capacity and demand rate of service provider $i$ are respectively

$$\mu_i^{POOL^*} = A_i - \frac{A_i\theta_i}{2(A_1\theta_1 + A_2\theta_2)} \left( \sqrt{M^2 + \Xi_{POOL}^2} + M \right) + \frac{1}{6} \left( \sqrt{M^2 + \Xi_{POOL}^2} - M \right) > 0,$$

$$\lambda_i^{POOL^*} = A_i - \frac{A_i\theta_i}{2(A_1\theta_1 + A_2\theta_2)} \left( \sqrt{M^2 + \Xi_{POOL}^2} + M \right) > 0,$$

and the total demand rate of the facility provider is

$$A^{POOL^*} = \frac{1}{2} \left[ A_1 + A_2 + K - \sqrt{M^2 + \Xi_{POOL}^2} \right],$$

where $M = A_1 + A_2 - K$ and $\Xi_{POOL} = \sqrt{24(A_1\theta_1 + A_2\theta_2)}$. 

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With the closed-form equilibrium in Theorem 1, we derive the comparative statics of service capacity levels and demand rates as the market conditions and facility capacity change.

**Proposition 2** If Assumption 1 holds, then: (1). Service provider $i$’s equilibrium service capacity $\mu_{i,\text{POOL}}^*$ and demand rate $\lambda_{i,\text{POOL}}^*$ are increasing in $K$, $A_i$ and $\theta_{i}$, but decreasing in $A_{-i}$ and $\theta_{-i}$; (2). The facility provider’s equilibrium total demand rate $\Lambda_{\text{POOL}}^*$ is increasing in $K$, $A_1$ and $A_2$, but decreasing in $\theta_1$ and $\theta_2$.

Proposition 2 shows that an increase of facility capacity ($K$) allows the service providers to increase their service capacity levels. As a result, the demand rate of each service provider and the total demand rate increase. A service provider increases its service capacity to attract customers as its potential market size ($A_i$) increases. This causes a drop in the available facility capacity for the other service provider and hence a reduction in the other service provider’s capacity and demand rate. However, the total demand rate increases. An increase of customers’ time sensitivity ($\theta_i$) drives down the customer demand rate and hence the required service capacity of service $i$. This increases the available facility capacity for the other service provider, who then increases its service capacity to attract more customers.

5 The Reservation Strategy

Suppose that the facility provider decides to allocate dedicated facility capacity $K_i$ to each service provider. This reservation strategy eliminates not only the gaming behavior of the service providers but also the pooling effect. Let $\Lambda_{i,\text{RES}}^*$ denote the total demand rate for the facility under this scenario.

By Proposition 1, if $K_i > 4\theta_i$, then service provider $i$ chooses service capacity $\mu_i^*(K_i)$, which brings in demand $\lambda_i^*(K_i)$ for the facility provider. If $K_i \leq 4\theta_i$, which implies that the dedicated facility capacity for service provider $i$ is too small, then service provider $i$ chooses to drop out of its market and not to use the facility. In this case, the dedicated facility generates no demand and is wasted. Hence, it is not optimal for the facility provider to allocate any dedicated capacity $K_i \in (0, 4\theta_i]$.

By Proposition 1, we have

$$\Lambda_{\text{RES}}(K_1, K_2) = \begin{cases} \sum_{i=1}^{2} \left\{ \frac{1}{2} (K_i + A_i) - \frac{1}{2} \sqrt{(K_i - A_i)^2 + 16A_i\theta_i} \right\}, & \text{if } K_i > 4\theta_i \text{ and } K_1 + K_2 = K; \\
\frac{1}{2} (K_i + A_i) - \frac{1}{2} \sqrt{(K - A_i)^2 + 16A_i\theta_i}, & \text{if } K_i = K \text{ and } K_{-i} = 0. \end{cases}$$

We let $K_{i,\text{RES}}^*$ be the optimal dedicated facility capacity for service provider $i$ and denote $\mu_{i,\text{RES}}^*$ and $\lambda_{i,\text{RES}}^*$ as the service capacity level and demand rate of service provider $i$, who is allocated facility capacity $K_{i,\text{RES}}^*$. The optimal total demand rate for the facility provider is $\Lambda_{\text{RES}}^* = \Lambda_{\text{RES}}(K_{1,\text{RES}}^*, K_{2,\text{RES}}^*)$.

We make the following assumption to avoid the trivial case that the facility provider causes one service provider to drop out.
Assumption 2 $A_i \geq 16\theta_i$, where $i = 1, 2$.

This assumption implies that customers are not extremely time sensitive and/or the potential market size is large.

Theorem 2 If Assumptions 1 and 2 hold, then the optimal facility capacity, the service capacity level of service provider $i$ and the demand rate are respectively

$$K_{RES}^* = A_i - \frac{\sqrt{A_i \theta_i}}{\sqrt{A_1 \theta_1} + \sqrt{A_2 \theta_2}} (A_1 + A_2 - K) > 0,$$

$$\mu_{RES}^* = A_i - \frac{\sqrt{A_i \theta_i}}{4(\sqrt{A_1 \theta_1} + \sqrt{A_2 \theta_2})} \left( \sqrt{M^2 + \frac{\Xi_{RES}^2}{\Xi_{RES}}} + 3M \right) > 0,$$

$$\lambda_{RES}^* = A_i - \frac{\sqrt{A_i \theta_i}}{2(\sqrt{A_1 \theta_1} + \sqrt{A_2 \theta_2})} \left( \sqrt{M^2 + \frac{\Xi_{RES}^2}{\Xi_{RES}}} + M \right) > 0,$$

and the total demand rate of the facility provider is

$$\Lambda_{RES}^* = \frac{1}{2} \left[ A_1 + A_2 + K - \sqrt{M^2 + \frac{\Xi_{RES}^2}{\Xi_{RES}}} \right],$$

where $\Xi_{RES} = 4 \left( \sqrt{A_1 \theta_1} + \sqrt{A_2 \theta_2} \right)$.

With the closed-form solutions in Theorem 2, we derive the comparative statics of the service capacity levels and demand rates as the market conditions and facility capacity change.

Proposition 3 If Assumptions 1 and 2 hold, then: (1). The optimal dedicated facility capacity $K_{RES}^*$ for service provider $i$, its service capacity $\mu_{RES}^*$ and demand rate $\lambda_{RES}^*$ are increasing in $K$ and $A_i$, but decreasing in $A_i$; (2). If $A_1 + A_2 \geq K$, then the optimal dedicated facility capacity $K_{RES}^*$ for service provider $i$, its service capacity $\mu_{RES}^*$ and demand rate $\lambda_{RES}^*$ are increasing in $\theta_{-i}$, but decreasing in $\theta_i$; (3). If $A_1 + A_2 < K$, then the optimal service capacity $\mu_{RES}^*$ of service provider $i$ and its demand rate $\lambda_{RES}^*$ are decreasing in $\theta_i$ and $\theta_{-i}$, but the optimal dedicated facility capacity $K_{RES}^*$ is increasing (decreasing) in $\theta_i$ ($\theta_{-i}$); (4). The facility provider’s total demand rate $\Lambda_{RES}^*$ is increasing in $K$, $A_1$ and $A_2$, but decreasing in $\theta_1$ and $\theta_2$.

Notice that most comparative statics in Proposition 3 are parallel to the ones in Proposition 2, except the comparative statics with respect to the time sensitivity parameters. Notice that $A_1 + A_2$ represents the total market size and $A_1 + A_2 > (\leq)K$ means that the facility provider’s capacity is (not) tight. In case that the facility provider’s capacity is tight (i.e., $A_1 + A_2 > K$), as the customer time-sensitivity $\theta_{-i}$ increases, service provider $i$’s market becomes relatively more attractive. Hence, the facility provider should allocate more facility capacity to service provider $i$ and decrease the dedicated capacity for service provider $-i$. As more facility capacity becomes available, service provider $i$ increases its service capacity to attract more customer demand. In
contrast, service provider $-i$ cuts its service capacity and loses demand, as its dedicated facility capacity is reduced.

In case that the facility provider’s capacity is not tight (i.e., $A_1 + A_2 < K$), as the customer time-sensitivity $\theta_{-i}$ increases, the facility provider adopts a very different strategy, that is, she moves facility capacity from service provider $i$ to service provider $-i$ and tries to keep the time-sensitive customers of service provider $-i$, even though this causes service provider $i$ to cut its service capacity and lose demand.

6 The Centralized System

We consider a centralized system in which the facility provider not only determines its own facility operations strategy but also the service capacity $\mu_i$ to maximize the total demand rate $\lambda_1 + \lambda_2$. This provides the first-best outcome for the overall system performance. Under the centralized system, the facility provider can operate its facility with two strategies: (1). By the reservation strategy, the facility provider allocates facility capacity $K_i$ to serve only service provider $i$, where $K_1 + K_2 = K$; (2). By the pooling strategy, the facility provider allows both service providers to share the entire facility.

Since the facility provider determines the service capacity level $\mu_i$, this eliminates the competition effect and the negative externality of the service providers’ gaming behavior demonstrated in Section 4. Hence, the pooling strategy is expected to dominate the reservation strategy. We establish this result in the following theorem.

**Theorem 3** It is always optimal for the facility provider to adopt the pooling strategy under the centralized system.

By Theorem 3, we focus on the pooling strategy. By this strategy, the facility provider maximizes $\Lambda_{\text{CEN}}(\mu_1, \mu_2) = \lambda_1(K - \mu_2, \mu_1) + \lambda_2(K - \mu_1, \mu_2)$, where $\lambda_i(K_1, \mu_i)$ is defined in Equation (2), $K_i = K - \mu_{-i}$ and $i = 1, 2$.

**Theorem 4** If Assumptions 1 and 2 hold, then the optimal service capacity and demand rate for service provider $i$ under the centralized system are respectively

$$\mu_{i}^{\text{CEN}} = A_i - \frac{1}{\Xi_{\text{CEN}}} \left[ \frac{A_i \theta_i}{\sqrt{A_i \theta_1 + A_2 \theta_2}} \left( M + \sqrt{M^2 + \Xi_{\text{CEN}}^2} \right) + 2 \sqrt{A_i \theta_i} \, M \right] > 0,$$

$$\lambda_{i}^{\text{CEN}} = A_i - \frac{1}{\Xi_{\text{CEN}}} \left( \frac{A_i \theta_i}{\sqrt{A_i \theta_1 + A_2 \theta_2}} + \sqrt{A_i \theta_i} \right) \left( M + \sqrt{M^2 + \Xi_{\text{CEN}}^2} \right) > 0,$$

and the optimal total demand rate of the facility provider is

$$\Lambda_{\text{CEN}} = \frac{1}{2} \left[ A_1 + A_2 + K - \sqrt{M^2 + \Xi_{\text{CEN}}^2} \right],$$

where $\Xi_{\text{CEN}} = 2 \left( \sqrt{A_1 \theta_1 + A_2 \theta_2} + \sqrt{A_1 \theta_1} + \sqrt{A_2 \theta_2} \right)$. 

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With the closed-form solutions in Theorem 4, we derive the comparative statics of the service capacity levels and demand rates as the market conditions and facility capacity change.

**Proposition 4** If Assumptions 1 and 2 hold, then: (1). Service provider \( i \)'s service capacity \( \mu_{CEN}^* \) and demand rate \( \lambda_{CEN}^* \) are increasing in \( K \); (2). The facility provider’s total demand rate \( \Lambda_{CEN}^* \) is increasing in \( K, A_1 \) and \( A_2 \), but decreasing in \( \theta_1 \) and \( \theta_2 \).

By Proposition 4, an increase in the facility capacity allows the facility provider to increase the service capacity levels and demand rates. Although the comparative statics of the total demand rate \( \Lambda_{CEN}^* \) with respect to the market conditions \( (A_i, \theta_i) \) are parallel to the ones in Propositions 2 and 3, the optimal service capacity \( \mu_{CEN}^* \) and individual demand rate \( \lambda_{CEN}^* \) may not be monotonic as the market conditions change. This is demonstrated in Example 1.

**Example 1** We let \( K = 650, A_1 = A_2 = 300 \) and \( \theta_1 = 5 \) and vary \( \theta_2 \in (0, 5] \). Figure 3 shows the optimal service capacity \( \mu_{CEN}^* \), the individual demand rate \( \lambda_{CEN}^* \) and the total demand rate \( \Lambda_{CEN}^* \). As customers become more time-sensitive to service 2, the facility provider initially increases the service capacity \( \mu_{CEN}^* \) to cut the dwell time, but the eventual demand rate decrease (in Figure 3(b)) reduces the required service capacity \( \mu_{CEN}^* \) (in Figure 3(a)). To shorten the facility time and the total processing time of service 2, the facility provider initially reduces service capacity \( \mu_{CEN}^* \) (in Figure 3(a)), which causes the demand rate of service 1 to drop slightly (in Figure 3(b)). But the eventual decrease of service capacity \( \mu_{CEN}^* \) makes more facility capacity available to service 1 and drives up service capacity \( \mu_{CEN}^* \) and demand rate \( \lambda_{CEN}^* \) (in Figures 3(a) and (b)). As shown in Figure 3(c), the total demand rate \( \Lambda_{CEN}^* \) is decreasing as the market conditions of service 2 deteriorate, which is consistent with Proposition 4.

Example 1 demonstrates that the service capacity decisions become much more complicated regard to the market condition changes if the decisions are jointly made for the best interest of
the whole system than if the decisions are separately made for the best interest of each individual service provider.

7 Comparisons and Managerial Insights

We compare the three scenarios studied in Sections 4-6. We let $\beta_i = A_i \theta_i$ for both $i = 1, 2$. Notice that $\beta_i$ represents the demand loss of service provider $i$ if the total processing time of service $i$ increases by one unit. In the example of port operations, a carrier with a large market size often has a large value of $\beta_i$. Furthermore, we define $\gamma_i = \beta_i / \beta_{-i}$ as the ratio of the two demand loss rates. Notice that $\gamma_1 = 1 / \gamma_2$.

First, we study the facility provider’s preference for the pooling and reservation strategies.

7.1 The Facility Provider’s Preference

We compare the total demand rates of the facility provider under the three scenarios in Sections 4-6, which are respectively $\Lambda^{POOL^*}$, $\Lambda^{RES^*}$ and $\Lambda^{CEN^*}$. The following theorem ranks them.

**Theorem 5** If Assumptions 1 and 2 hold, then: (1). $\Lambda^{CEN^*} > \Lambda^{POOL^*}$ and $\Lambda^{CEN^*} > \Lambda^{RES^*}$; (2). When $7 - 4\sqrt{3} < \gamma_1 < 7 + 4\sqrt{3}$, $\Lambda^{RES^*} < \Lambda^{POOL^*}$; (3). When $\gamma_1 = 7 \pm 4\sqrt{3}$, $\Lambda^{RES^*} = \Lambda^{POOL^*}$; (4). When $\gamma_1 < 7 - 4\sqrt{3}$ or $\gamma_1 > 7 + 4\sqrt{3}$, $\Lambda^{RES^*} > \Lambda^{POOL^*}$.

From the facility provider’s viewpoint, using capacity reservation eliminates the competition effect between the service providers, but it also eliminates the pooling effect. If the negative externality of the service providers’ gaming behavior offsets the pooling benefit, the facility provider should adopt the reservation strategy. Theorem 5 gives a clear criterion on this tradeoff, which only depends on the demand loss ratio $\gamma_1$. It is better for the facility provider to adopt the reservation strategy if and only if the demand loss ratio is significantly different from 1, i.e., $\gamma_1 > 7 + 4\sqrt{3} \approx 14$ or $\gamma_1 < 7 - 4\sqrt{3} \approx 1/14$. The condition means that the demand characteristics of the two service providers must be significantly different. When $\theta_1$ and $\theta_2$ are similar, one service provider has to be dominant so that the facility provider adopts the reservation strategy. This result is supported by the real practice in port operations. As reported by the Port Reform Toolkit (The World Bank 2007), ports dominated by one carrier often provide dedicated facilities to the dominant carrier, but ports that serve equally sized carriers are unwilling to provide dedicated facilities. For instance, the Maersk Line accounts for almost 80%-90% of the traffic at the Port of Salalah, and it is much larger than other carriers using the port. Hence, it is not surprising that the port provides the Maersk Line with dedicated facilities. On the other hand, at the ports of Singapore, Hong Kong and Shanghai, which are the three largest ports in the world, no carrier is dominant over the others. Hence, these ports do not provide dedicated facilities (The World Bank 2007, Module 3, Page 86).
When to pool separate subsystems together has been studied in the queueing literature. Pangburn and Stavrulaki (2008), Smith and Whitt (1981), Whitt (1999) and Yu et al. (2009) found that pooling is not optimal if customer characteristics (e.g., service time distributions and time sensitivity) are significantly different. This observation is confirmed in our model. One of the reasons of not pooling is that the service providers have significantly different demand loss rates, which implies that the pooling benefit is low. However, unlike the previous literature, we identify another reason of not pooling, that is, pooling self-interested service providers introduces facility capacity competition and creates facility over-utilization and congestion. The congestion effect can be measured by the total traffic (i.e., the total service capacity level) that uses the facility.

**Theorem 6** If Assumptions 1 and 2 hold, then \( \mu_{\text{POOL}} > \mu_{\text{CEN}} > \mu_{\text{RES}} \), where \( \mu_{\text{POOL}} = \mu_{\text{POOL}}^1 + \mu_{\text{POOL}}^2 \), \( \mu_{\text{RES}} = \mu_{\text{RES}}^1 + \mu_{\text{RES}}^2 \) and \( \mu_{\text{CEN}} = \mu_{\text{CEN}}^1 + \mu_{\text{CEN}}^2 \).

Theorem 6 suggests that the facility is always in the busiest mode if the facility provider pools the service providers together. On the positive side, the pooling strategy makes the facility accessible for both service providers, which stimulates the total traffic amount and increases the facility utilization rate (i.e., \( \mu_{\text{POOL}} > \mu_{\text{RES}} \) and \( \mu_{\text{CEN}} > \mu_{\text{RES}} \)). However, on the negative side, the pooling strategy allows the service providers to compete on the facility capacity, which makes the facility overused and creates congestion (i.e., \( \mu_{\text{POOL}} > \mu_{\text{CEN}} \)).

Since congestion causes delays, lowers service quality and hence hurts the demand rates, the negative effect of the facility capacity competition may offsets the pooling benefit. This is why the facility provider may not prefer the pooling strategy. Notice that if the facility provider can eliminate the facility capacity competition between the service providers (e.g., in the centralized system), then pooling is always optimal. Hence, facility capacity competition is a prerequisite condition for not pooling the service providers together.

Second, we study the service providers’ preferences for the capacity reservation and pooling strategies.

### 7.2 The Service Providers’ Preference

We compare the demand rates of service provider \( i \) under the two scenarios in Sections 4-5, which are respectively \( \lambda_{\text{POOL}}^i \) and \( \lambda_{\text{RES}}^i \). The following theorem ranks them.

**Theorem 7** If Assumptions 1 and 2 hold, then there exists a threshold \( \gamma_i \in (1, 4) \) such that \( \lambda_{\text{POOL}}^i > \lambda_{\text{RES}}^i \) if \( \gamma_i < \gamma_i \) and \( \lambda_{\text{POOL}}^i < \lambda_{\text{RES}}^i \) if \( \gamma_i > \gamma_i \).

By Theorem 7, if the demand loss ratio is smaller than 1, then the service provider with the smaller demand loss rate prefers to sharing the facility capacity with the other service provider, since the benefit of accessing the whole facility for the smaller service provider dominates the
negative externality generated by the larger service provider. In contrast, if the demand loss ratio is larger than 4, then the service provider with the larger demand loss rate prefers to using capacity reservation. This explains why large carriers often ask for dedicated facilities. However, their requests may be denied by a port authority if their market sizes are not sufficiently dominant (i.e., the demand loss ratio is less than 14 in Theorem 5).

Figure 4 summarizes the facility provider’s and service providers’ preferences for the capacity reservation and pooling strategies under different market conditions.

Finally, we consider a numerical example.

**Example 2** We let $K = 125$, $A_1 = 100$, $\theta_1 = 1$ and $\theta_2 = 0.25$ and vary $A_2 \in [20, 100]$. Figure 5 shows the optimal demand rates $\Lambda^I$, $\lambda^I_1$ and $\lambda^I_2$, and service capacity levels $\mu^I$, $\mu^I_1$ and $\mu^I_2$, where $I \in \{ \text{POOL, RES, CEN} \}$.

As shown in Figure 5(a), if the market size of service provider 2 is very small (i.e., $A_2 < 30$), the optimal total demand rate $\Lambda_{\text{RES}}^*$ is higher than $\Lambda_{\text{POOL}}^*$, which implies that the facility provider prefers the reservation strategy. Otherwise, $\Lambda_{\text{RES}}^* < \Lambda_{\text{POOL}}^*$, which implies that the facility provider prefers the pooling strategy. This is consistent with Theorem 5. Figure 5(b) shows that the total traffic rate $\mu_{\text{POOL}}^*$ under the facility competition is the highest among the three scenarios (i.e., pooling, reservation and centralization). This is consistent with Theorem 6.

Notice that service provider 1 has a larger market size and is more time-sensitive than service provider 2, which implies that service provider 1 has a larger demand loss rate than service provider 2. As shown in Figures 5(c) and (e), service provider 1 prefers the reservation strategy since $\lambda_{\text{RES}}^* > \lambda_{\text{POOL}}^*$, but service provider 2 prefers the pooling strategy since $\lambda_{\text{RES}}^* < \lambda_{\text{POOL}}^*$. This is consistent with Theorem 7.

Since service provider 2 is less time-sensitive than service provider 1, the facility competition and its congestion consequence has less negative effect on service provider 2 than on service provider 1. Hence, service provider 2 behaves more aggressively under the facility competition than service provider 1. This causes that service provider 2’s optimal service capacity $\mu_{\text{POOL}}^*$ is the highest among the three scenarios (as shown in Figure 5(f)), but service provider 1’s optimal service capacity $\mu_{\text{POOL}}^*$
Figure 5: (a). Optimal total demand rate $\Lambda^I_*$; (b). Optimal total service capacity $\mu^I_*$; (c). Optimal demand rate of service provider 1 $\lambda^I_1$; (d). Optimal service capacity of service provider 1 $\mu^I_1$; (e). Optimal demand rate of service provider 2 $\lambda^I_2$; (f). Optimal service capacity of service provider 2 $\mu^I_2$, where $I \in \{\text{POOL, RES, CEN}\}$. 
is the lowest among the three scenarios (as shown in Figure 5(d)). Hence, the monotonic ranking of the total traffic rate in Theorem 6 may be reversed at the individual service provider level.

Finally, as shown in Figure 5, the optimal demand rates and service capacity levels are monotonic in $A_2$, which is consistent with Propositions 2-4.

8 Concluding Remarks

In this paper, we consider a facility provider that offers its facilities to two service providers, who determine their service capacity levels to serve two separate markets. The facility provider can either pool the service providers together and let them share the facilities or allocate a dedicated facility to the service providers and serve them separately. We assume that the demand rate of a service is linearly decreasing in the total processing time of the service, which is a function of the service capacity and facility capacity levels.

We find that the choice between the pooling and reservation strategies critically depends on the ratio of the demand loss rates of the service providers. The facility provider prefers the reservation strategy to the pooling strategy if one service provider’s demand loss rate is much larger than the other’s. In contrast, if the demand loss rates of the service providers are close, the facility provider prefers the pooling strategy. The service providers’ preferences are different from the facility provider’s. The dominant service provider, whose demand loss rate is four times larger than the other’s, prefers dedicated facilities. In contrast, the smaller service provider, whose demand loss rate is less than the other’s, prefers the pooling strategy.

We also study a centralized system, in which the facility provider fully controls the service providers and hence is able to eliminate the negative effect of the facility competition between the service providers. We find that pooling is always optimal for the facility provider, which suggests that facility capacity competition is a prerequisite condition for not pooling the service providers.

There are three directions in which this research could be extended. First, we assume that the service providers maximize their demand rates, as cargo volume is one of the most important performance measures in the maritime industry. Another important performance measure for service providers is profit. It is an interesting research question to design a joint optimal pricing policy and a capacity management strategy for a port authority who deals with profit-maximizing carriers. Second, we assume that there are two service providers, which is sufficient to show the tradeoff between the capacity reservation and pooling strategies. But, in practice, a port often serves multiple carriers, who may benefit from forming alliances to share dedicated port facilities. This research question can be studied under a cooperative game framework. Finally, seaports face heavy competition from local competitors. Capacity reservation can be used as a strategic weapon to attract the business of carriers. A game theoretical model can be formed to study competition among seaports.
Appendix

Derivation of Equation (2). By replacing \( t_i \), we have \( \frac{1}{v^2_i} (1 - \frac{\lambda_i}{K_i^*}) = \frac{1}{\kappa_i - \mu_i} + \frac{1}{\mu_i} \). Notice that the left side of this equation is decreasing in \( \lambda_i \), but the right side is increasing in \( \lambda_i \). Hence, the equation has a solution in \( (0, A_i) \) if and only if \( \frac{1}{\delta_i} > \frac{1}{\kappa_i - \mu_i} + \frac{1}{\mu_i} \), which is equivalent to \( K_i > 4\theta_i \) and \( \frac{1}{2} \left( K_i - \sqrt{K_i^2 - 4K_i^*} \right) < \mu_i < \frac{1}{2} \left( K_i + \sqrt{K_i^2 - 4K_i^*} \right) \). By rearranging the terms of the equation, we have the quadratic equation \( \lambda_i^2 - (A_i + \mu_i - \frac{4\theta_i}{K_i^*})\lambda_i + A_i\mu_i - A_i\theta_i - \frac{4\theta_i\mu_i}{K_i^*} = 0 \). Notice that the quadratic equation takes a negative value at \( \lambda_i = \mu_i \) and a positive value at \( \lambda_i = 0 \). Hence, we take the smaller root. This implies Equation (2). □

Proof of Proposition 1. For Claim 1, notice that \( \frac{\partial \lambda_i}{\partial \mu_i} = -\frac{A_i\theta_i}{(K_i^* - \mu_i)^2} - 1 \). Then, by some algebra, we have

\[
\frac{\partial \lambda_i}{\partial \mu_i} = 1 - \frac{1}{2} \left( \frac{A_i\theta_i}{(K_i^* - \mu_i)^2} + 1 \right) \left( 1 - \frac{\delta_i}{\sqrt{\delta_i^2 + 4A_i\theta_i}} \right) .
\]

To simplify the notation, we let \( \beta_i = A_i\theta_i \) and \( v_i = K_i - \mu_i \). Then

\[
\frac{\partial \lambda_i}{\partial \mu_i} = 1 - \frac{1}{2} \left( \frac{\beta_i}{v_i^2} + 1 \right) \left( 1 - \frac{\delta_i}{\sqrt{\delta_i^2 + 4\beta_i}} \right) .
\]

Letting \( \frac{\partial \lambda_i}{\partial \mu_i} = 0 \), we have \( \frac{2v_i^2}{\beta_i + \delta_i^2} = 1 - \frac{\delta_i}{\sqrt{\delta_i^2 + 4\beta_i}} \). By some algebra, this equation becomes \( \delta_i^2 + 4\beta_i = \left( \frac{\beta_i}{v_i} + v_i \right)^2 \). By the definition of \( \delta_i \), we have \( \delta_i = A_i - \frac{\beta_i}{v_i} - K_i + v_i \). Then

\[
\left( \frac{\beta_i}{v_i} + v_i \right)^2 = \left( A_i - \frac{\beta_i}{v_i} - K_i + v_i \right)^2 + 4\beta_i = (A_i - K_i)^2 - 2(A_i - K_i) \left( \frac{\beta_i}{v_i} - v_i \right) + \left( \frac{\beta_i}{v_i} + v_i \right)^2.
\]

Hence, \( \frac{\beta_i}{v_i} - v_i = \frac{1}{2}(A_i - K_i) \). Since \( v_i = K_i - \mu_i > 0 \), we have \( v_i = -\frac{1}{4}(A_i - K_i) + \frac{1}{2} \sqrt{(A_i - K_i)^2 + 16\beta_i} \). Hence, the first order condition of \( \lambda_i(K_i, \mu_i) \) has a unique solution. Since \( \lambda_i(K_i, \mu_i) = 0 \) for \( \mu_i \notin \left[ \frac{1}{2} K_i - \sqrt{K_i^2 - 4K_i^*} \right], \frac{1}{2} \left( K_i + \sqrt{K_i^2 - 4K_i^*} \right) \right] \), \( \lambda_i(K_i, \mu_i) \) is quasi-concave in \( \mu_i \). Hence, the optimal service capacity level is

\[
\mu_i^*(K_i) = K_i - v_i = \frac{A_i + 3K_i}{4} - \frac{1}{4} \sqrt{(A_i - K_i)^2 + 16A_i\theta_i}.
\]

For Claim 2, notice that \( \delta_i^2 + 4\beta_i = \left( \frac{\beta_i}{v_i} + v_i \right)^2 = \left( \frac{\beta_i}{v_i} - v_i \right)^2 + 4\beta_i = \frac{1}{4}(A_i - K_i)^2 + 4\beta_i \). By Equation (2), the optimal demand rate

\[
\lambda_i^*(K_i) = \frac{1}{2} \left( A_i - \frac{\beta_i}{K_i - \mu_i^*(K_i)} + \mu_i^*(K_i) - \sqrt{\delta_i^2 + 4\beta_i} \right)
\]

\[
= \frac{1}{2} \left( A_i + K_i - \frac{\beta_i}{v_i} - v_i - \sqrt{\delta_i^2 + 4\beta_i} \right) = \frac{A_i + K_i}{2} - \frac{1}{2} \sqrt{(A_i - K_i)^2 + 16A_i\theta_i}.
\]
For Claim 3, since \( \mu_i^*(K_i) - \lambda_i^*(K_i) = \frac{K_i - A_i}{4} + \frac{1}{4} \sqrt{(A_i - K_i)^2 + 16A_i \theta_i} > 0 \), \( \mu_i^*(K_i) > \lambda_i^*(K_i) \). Furthermore, since \( K_i > 4A_i \), \( 16A_i \theta_i < 4A_i K_i \) and \( (A_i - K_i)^2 + 16A_i \theta_i < A_i + K_i \). Hence, the optimal demand rate \( \lambda_i^*(K_i) > 0 \).

For Claim 4, we see that

\[
\frac{\partial \lambda_i^*}{\partial K_i} = \frac{1}{2} \left[ 1 - \frac{K_i - A_i}{\sqrt{(A_i - K_i)^2 + 16A_i \theta_i}} \right] > 0,
\]
\[
\frac{\partial \lambda_i^*}{\partial A_i} = \frac{1}{2} \left[ 1 - \frac{(A_i - K_i) + 8 \theta_i}{\sqrt{(A_i - K_i)^2 + 16A_i \theta_i}} \right] \geq \frac{1}{2} \left[ 1 - \frac{(A_i - K_i)^2 + 16 \theta_i (A_i + 4 \theta_i - K_i)}{(A_i - K_i)^2 + 16 \theta_i A_i} \right] > 0,
\]
and \( \frac{\partial \lambda_i^*}{\partial \theta_i} < 0 \). Notice that \( \mu_i^*(K_i) = \frac{K_i + \lambda_i^*(K_i)}{2} \). Hence, Claim 4 holds.

\[\square\]

**Proof of Theorem 1.** By Assumption 1, the condition of Proposition 1 always holds at any equilibrium. Then by Claims 1 and 2 of Proposition 1, the equilibrium service capacity level \( \mu_i^{\text{POOL}} \) and demand rate \( \lambda_i^{\text{POOL}} \) must satisfy the following equations:

\[
\mu_i^{\text{POOL}} = \frac{3(K - \mu_i^{\text{POOL}}) + A_i}{4} - \frac{1}{4} \sqrt{(K - \mu_i^{\text{POOL}} - A_i)^2 + 16A_i \theta_i},
\]
\[
\lambda_i^{\text{POOL}} = \frac{(K - \mu_i^{\text{POOL}}) + A_i}{2} - \frac{1}{2} \sqrt{(K - \mu_i^{\text{POOL}} - A_i)^2 + 16A_i \theta_i},
\]
for both \( i = 1, 2 \).

Let \( w_i = A_i - K + \mu_{-i} \) and \( \beta_i = A_i \theta_i \). By Equation (3), we have

\[
\mu_i = w_{-i} - A_{-i} + K = \frac{A_i + 3(A_i - w_i)}{4} - \frac{1}{4} \sqrt{w_i^2 + 16 \beta_i}.
\]

Then we have \( w_{-i} = M - \frac{3}{4} w_i - \frac{1}{4} \sqrt{w_i^2 + 16 \beta_i} \). Let \( \Delta = w_1 + w_2 \). Then \( \Delta = M + \frac{1}{4} w_i - \frac{1}{4} \sqrt{w_i^2 + 16 \beta_i} \).

Solving the equation, we have

\[
w_i = \frac{2 \beta_i}{M - \Delta} - 2(M - \Delta).
\]

Since \( \Delta = w_1 + w_2 \), we have

\[
\Delta = \frac{2(\beta_1 + \beta_2)}{M - \Delta} - 4(M - \Delta).
\]

Let \( \Sigma = M - \Delta \). Notice that \( \Delta = w_1 + w_2 = A_1 + A_2 - 2K + \mu_1 + \mu_2 \). Then \( \Sigma = M - \Delta = K - \mu_1 - \mu_2 > 0 \). By Equation (7), we have \( 3 \Sigma + M - \frac{2(\beta_1 + \beta_2)}{\Sigma} = 0 \). Solving the equation for \( \Sigma \), since \( \Sigma > 0 \), we have \( \Sigma = \frac{1}{6} \left( \sqrt{M^2 + \Xi_{\text{POOL}}^2} - M \right) \).

By Equation (6), \( w_i = \frac{2 \beta_i}{M - \Delta} - 2 \Sigma \). Then \( \sqrt{w_i^2 + 16 \beta_i} = \sqrt{\left(\frac{2 \beta_i}{\Sigma} - 2 \Sigma\right)^2 + 16 \beta_i} = \frac{2 \beta_i}{\Sigma} + 2 \Sigma \). By Equation (5),

\[
\mu_i^{\text{POOL}} = A_i - \frac{3}{4} w_i - \frac{1}{4} \sqrt{w_i^2 + 16 \beta_i} = A_i - \frac{2 \beta_i}{\Sigma} + \Sigma
\]

\[
= A_i - \frac{\beta_i}{2(\beta_1 + \beta_2)} \left( \sqrt{M^2 + \Xi_{\text{POOL}}^2} + M \right) + \frac{1}{6} \left( \sqrt{M^2 + \Xi_{\text{POOL}}^2} + M \right).
\]

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Then by Equation (4),
\[
\lambda_i^\text{POOL*} = \frac{1}{2} (A_i + K - \mu_{i-1}^\text{POOL*}) - \frac{1}{2} \sqrt{w_i^2 + 16 \beta_i} = A_i - \frac{w_i}{2} - \frac{1}{2} \sqrt{w_i^2 + 16 \beta_i}
\]
\[
= A_i - \frac{2 \beta_i}{\Sigma} = A_i - \frac{\beta_i}{2(\beta_1 + \beta_2)} \left( \sqrt{M^2 + \Xi^2_{\text{POOL}} + M} \right).
\]

Finally, we have
\[
\Lambda^\text{POOL*} = \lambda_1^\text{POOL*} + \lambda_2^\text{POOL*} = A_1 + A_2 - \frac{1}{2} \left( \sqrt{M^2 + \Xi^2_{\text{POOL}} + M} \right) = \frac{1}{2} \left[ A_1 + A_2 + K - \sqrt{M^2 + \Xi^2_{\text{POOL}}} \right].
\]

This concludes the proof of the theorem.

**Proof of Proposition 2.** Without loss of generality, we let \( i = 1 \). Notice that
\[
\lambda_1^\text{POOL*} = A_1 - \frac{A_1 \theta_1}{2(A_1 \theta_1 + A_2 \theta_2)} \left( \sqrt{M^2 + \Xi^2_{\text{POOL}} + M} \right) = A_1 \left( 1 - \frac{12 \theta_1}{\sqrt{M^2 + \Xi^2_{\text{POOL}}} - M} \right) > 0.
\]

Then \( \frac{\partial \lambda_i^\text{POOL*}}{\partial A} = 12A_1 \theta_1 \left( 1 - M/\sqrt{M^2 + \Xi^2_{\text{POOL}}} \right) / \left( \sqrt{M^2 + \Xi^2_{\text{POOL}}} - M \right)^2 > 0, \)
\[
\frac{\partial \lambda_i^\text{POOL*}}{\partial A_1} = \left( 1 - \frac{12 \theta_1}{\sqrt{M^2 + \Xi^2_{\text{POOL}}} - M} \right) + 12A_1 \theta_1 \left( \frac{M + 12 \theta_1}{\sqrt{M^2 + \Xi^2_{\text{POOL}}} - M} - 1 \right)
\]
\[
= \left( 1 - \frac{12 \theta_1}{\sqrt{M^2 + \Xi^2_{\text{POOL}}} - M} \right) \left[ 1 - \frac{12A_1 \theta_1}{\sqrt{M^2 + \Xi^2_{\text{POOL}}} \left( \sqrt{M^2 + \Xi^2_{\text{POOL}}} - M \right)} \right] \tag{8}
\]
\[
= \frac{\lambda_i^\text{POOL*}}{A_1} \left[ 1 - \frac{A_1 \theta_1}{2(A_1 \theta_1 + A_2 \theta_2)} \left( 1 + \frac{M}{\sqrt{M^2 + \Xi^2_{\text{POOL}}} \sqrt{M^2 + \Xi^2_{\text{POOL}}} - M} \right) \right] \tag{9}
\]
\[
> \frac{\lambda_i^\text{POOL*}}{A_1} \left[ 1 - \frac{A_1 \theta_1}{A_1 \theta_1 + A_2 \theta_2} \right] > 0,
\]
\[
\frac{\partial \lambda_i^\text{POOL*}}{\partial A_2} = 12A_1 \theta_1 \left( \frac{M + 12 \theta_1}{\sqrt{M^2 + \Xi^2_{\text{POOL}}} - M} - 1 \right) - \frac{12A_1 \theta_1}{\sqrt{M^2 + \Xi^2_{\text{POOL}}} \left( \sqrt{M^2 + \Xi^2_{\text{POOL}}} - M \right)} \frac{\lambda_i^\text{POOL*}}{A_2} < 0,
\]
\[
\frac{\partial \lambda_i^\text{POOL*}}{\partial \theta_2} = 12A_1 \theta_1 \cdot 12A_2 \left[ \sqrt{M^2 + \Xi^2_{\text{POOL}}} \left( \sqrt{M^2 + \Xi^2_{\text{POOL}}} - M \right) \right] > 0,
\]
and by Equation (8), \( \frac{\partial \lambda_i^\text{POOL*}}{\partial \theta_1} = - \frac{12A_1}{\sqrt{M^2 + \Xi^2_{\text{POOL}}} - M} \left[ 1 - \frac{12A_1 \theta_1}{\sqrt{M^2 + \Xi^2_{\text{POOL}}} \left( \sqrt{M^2 + \Xi^2_{\text{POOL}}} - M \right)} \right] < 0. \) Hence, \( \lambda_i^\text{POOL*} \) is increasing in \( K, A_i \) and \( \theta_{-i} \) but decreasing in \( A_{-i} \) and \( \theta_i \).
Notice that $\mu_{pool}^i = \lambda_{pool}^i + \frac{1}{6} \left( \sqrt{M^2 + \Xi_{pool}} - M \right)$. Then \( \frac{\partial \mu_{pool}^i}{\partial K} = \frac{\partial \lambda_{pool}^i}{\partial K} + \frac{1}{6} \left( 1 - \frac{M}{\sqrt{M^2 + \Xi_{pool}}} \right) > 0 \). By Equation (9),

\[
\frac{\partial \mu_{pool}^i}{\partial A_1} = \frac{\lambda_{pool}^i}{A_1} \left[ 1 - \frac{A_1 \theta_i}{2(A_1 \theta_i + A_2 \theta_2)} \right] \left( 1 + \frac{M}{\sqrt{M^2 + \Xi_{pool}}} \right) + \frac{1}{6} \left[ \frac{M + 12\theta_i}{\sqrt{M^2 + \Xi_{pool}}} - 1 \right]
\]

\[
\frac{\partial \mu_{pool}^i}{\partial A_2} = \frac{\lambda_{pool}^i}{A_2} \left[ 2 - \frac{A_1 \theta_i}{2(A_1 \theta_i + A_2 \theta_2)} \right] \left( 1 + \frac{M}{\sqrt{M^2 + \Xi_{pool}}} \right) + \frac{1}{6} \left( \frac{M}{\sqrt{M^2 + \Xi_{pool}}} \right) > 0
\]

Similarly, we have \( \frac{\partial \mu_{pool}^i}{\partial \theta_1} = \frac{\lambda_{pool}^i}{\theta_1} - \frac{1}{6} \frac{A_1}{M} \left( 1 - M/\sqrt{M^2 + \Xi_{pool}} \right) < 0 \). Furthermore, by Equations (8)-(10),

\[
\frac{\partial \mu_{pool}^i}{\partial \theta_2} = \frac{\lambda_{pool}^i}{\theta_2} + 2A_2/\sqrt{M^2 + \Xi_{pool}} > 0
\]

and \( \frac{\partial \mu_{pool}^i}{\partial A_i} = \frac{\lambda_{pool}^i}{A_i} \left( 1 - M/\sqrt{M^2 + \Xi_{pool}} \right) > 0 \). Hence, \( \mu_{pool}^i \) is increasing in \( K, A_i \) and \( \theta_1 \), but decreasing in \( A_i \) and \( \theta_1 \).

Since \( \Xi_{pool} \) is increasing in \( \theta_i \), \( \Lambda_{pool}^i \) is decreasing in \( \theta_i \). Furthermore, \( \frac{\partial \lambda_{pool}^i}{\partial K} = \frac{1}{2} \left( 1 + \frac{M}{\sqrt{M^2 + \Xi_{pool}}} \right) > 0 \) and

\[
\frac{\partial \lambda_{pool}^i}{\partial A_i} = \frac{1}{2} \left( 1 - \frac{M + 12\theta_i}{\sqrt{M^2 + \Xi_{pool}}} \right) = \frac{\lambda_{pool}^i}{2A_i} \left( 1 - M/\sqrt{M^2 + \Xi_{pool}} \right) > 0
\]

for both \( i = 1, 2 \). Hence, \( \Lambda_{pool}^i \) is increasing in \( K, A_i \) and \( A_2 \) but decreasing in \( \theta_1 \) and \( \theta_2 \).

**Proof of Theorem 2.** First, we maximize \( \sum_{i=1}^2 \frac{1}{2} (A_i + K_i) - \frac{1}{2} \sqrt{(A_i - K_i)^2 + 16A_i} \) subject to \( K_1 + K_2 = K \).

Let \( \tau \) be the Lagrange multiplier and \( \beta_i = A_i \theta_i \). Then the Lagrange function is

\[
g(K_1, K_2, \tau) = \sum_{i=1}^2 \frac{1}{2} (A_i + K_i) - \frac{1}{2} \sqrt{(A_i - K_i)^2 + 16\beta_i} - \tau(K_1 + K_2 - K).
\]

Setting \( \partial g/\partial K_i = 0 \), we have \( A_i - K_i = (2\tau - 1) \sqrt{(A_i - K_i)^2 + 16\beta_i} \). Then we have \( A_i - K_i = 4\sqrt{\beta_i} (2\tau - 1)/\sqrt{1 - (2\tau - 1)^2} \), and

\[
K_i = A_i - 4\sqrt{\beta_i} (2\tau - 1)/\sqrt{1 - (2\tau - 1)^2}.
\]

(11)

Since \( K_1 + K_2 = K \), we have \( K = A_1 + A_2 - 4 (\sqrt{\beta_1} + \sqrt{\beta_2}) (2\tau - 1)/\sqrt{1 - (2\tau - 1)^2} \). Hence, \( (2\tau - 1)/\sqrt{1 - (2\tau - 1)^2} = M/ \left[ 4 \left( \sqrt{\beta_1} + \sqrt{\beta_2} \right) \right] \). By Equation (11), we have

\[
K_i^* = A_i - \frac{\sqrt{\beta_i}}{\sqrt{\beta_1} + \sqrt{\beta_2}} M = A_i - \frac{\sqrt{\beta_i}}{\sqrt{\beta_1} + \sqrt{\beta_2}} (A_1 + A_2 - K_i).
\]

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By Proposition 1, we have

\[ \lambda_i^* = \frac{1}{2}(A_i + K_i^*) - \frac{1}{2}\sqrt{(A_i - K_i^*)^2 + 16\beta_i} = A_i - \frac{\sqrt{A_i\theta_i}}{2(\sqrt{A_i\theta_i} + \sqrt{A_i\theta_2})} \left( \sqrt{M^2 + \Xi^2_{\text{RES}}} + M \right), \]

\[ \mu_i^* = \frac{1}{2}(K_i^* + \lambda_i^*) = A_i - \frac{\sqrt{A_i\theta_i}}{4(\sqrt{A_i\theta_1} + \sqrt{A_i\theta_2})} \left( \sqrt{M^2 + \Xi^2_{\text{RES}}} + 3M \right), \]

and \( \lambda_1^* + \lambda_2^* = \frac{1}{2} \left[ A_1 + A_2 + K - \sqrt{M^2 + \Xi^2_{\text{RES}}} \right]. \)

Second, we prove that \( K_1^* > 4\theta_i \). Without loss of generality, we let \( i = 1 \). Notice that

\[ K_1^* = A_1\sqrt{\beta_1} + A_1\sqrt{\beta_2} - A_1\sqrt{\beta_1} - A_2\sqrt{\beta_1} + K\sqrt{\beta_1} = \frac{A_1\sqrt{\beta_2} - A_2\sqrt{\beta_1} + K\sqrt{\beta_1}}{\sqrt{\beta_1} + \sqrt{\beta_2}}. \]

Since \( A_1 \geq 16\theta_1 \) and \( K \geq \max(A_1, A_2) + 8\max(\theta_1, \theta_2) \), \( K_1^* \geq (16\theta_1\sqrt{\beta_2} + 8\theta_1\sqrt{\beta_1}) / (\sqrt{\beta_1} + \sqrt{\beta_2}) > 4\theta_1 \). Hence, there exists a unique candidate \( (K_1^*, K_2^*) \), which satisfies the first order condition and the constraints \( K_1^* + K_2^* = K \) and \( K_1^* > \theta_i \) for both \( i = 1, 2 \). Since \( \Lambda_{\text{RES}}(4\theta_1, K - 4\theta_1) < \lambda_2^*(K) \) and \( \Lambda_{\text{RES}}(K - 4\theta_2, 4\theta_2) < \lambda_1^*(K), (K_1^*, K_2^*) \) is the optimal solution as long as we establish the next claim.

Third, we prove \( \lambda_1^* + \lambda_2^* > \lambda_i^*(K) \) for both \( i = 1, 2 \). Without loss of generality, we let \( i = 1 \). Since \( \lambda_1^* + \lambda_2^* = \frac{1}{2} \left[ A_1 + A_2 + K - \sqrt{(K - A_1 - A_2)^2 + 16(\sqrt{\beta_1} + \sqrt{\beta_2})^2} \right] \) and \( \lambda_i^*(K) = \frac{1}{2} \left[ A_1 + A_2 + K - \sqrt{(K - A_1)^2 + 16\beta_1} \right] \), it is equivalent to prove that \( A_2 + \sqrt{(K - A_1)^2 + 16\beta_1} > \sqrt{(K - A_1 - A_2)^2 + 16(\sqrt{\beta_1} + \sqrt{\beta_2})^2} \). Taking square on both sides of the inequality and by some algebra, we find that it is equivalent to prove

\[ A_2\sqrt{(K - A_1)^2 + 16\beta_1} + A_2(K - A_1) > 8\beta_2 + 16\sqrt{\beta_1}\beta_2. \]  

Since \( A_i \geq 16\theta_i \) for both \( i = 1, 2 \), \( A_2 \geq \sqrt{A_2\sqrt{16\theta_2}} = 4\beta_2 \). Furthermore, \( K \geq \max(A_1, A_2) + 8\max(\theta_1, \theta_2) \). Then \( A_2(K - A_1) \geq 8A_2\theta_2 = 8\beta_2 \). Hence, LHS of Inequality (12) \( > A_2\sqrt{\beta_1} + A_2(K - A_1) \geq 16\sqrt{\beta_1}\beta_2 + 8\beta_2 = \text{RHS of Inequality (12)} \). Hence, \( \lambda_1^* + \lambda_2^* > \lambda_i^*(K) \) for both \( i = 1, 2 \).

Hence, \( K_{i, \text{RES}}^* = K_i^*, \mu_i^{\text{RES}} = \mu_i^*, \lambda_i^{\text{RES}} = \lambda_i^* \), and \( \Lambda_{\text{RES}} = \lambda_1^* + \lambda_2^* \).

**Proof of Proposition 3.** First, we derive the comparative statics of \( \Lambda_{\text{RES}} \). Since \( \Xi_{\text{RES}} \) is increasing in \( \theta_i \), \( \Lambda_{\text{RES}} \) is decreasing in \( \theta_i \). Furthermore, \( \frac{\partial \Lambda_{\text{RES}}}{\partial A_i} = \frac{1}{2} \left( 1 + M/\sqrt{M^2 + \Xi^2_{\text{RES}}} \right) > 0 \) and

\[ \frac{\partial \Lambda_{\text{RES}}}{\partial A_i} = \frac{1}{2} \left( 1 - \frac{M + 2\sqrt{\theta_i/A_i\Xi_{\text{RES}}}}{\sqrt{M^2 + \Xi^2_{\text{RES}}}} \right) = \frac{\sqrt{M^2 + \Xi^2_{\text{RES}}} - M - 2\sqrt{\theta_i/A_i\Xi_{\text{RES}}}}{2\sqrt{M^2 + \Xi^2_{\text{RES}}}} = \frac{\Xi^2_{\text{RES}} - 2\sqrt{\theta_i/A_i\Xi_{\text{RES}}(\sqrt{M^2 + \Xi^2_{\text{RES}}} + M)}}{2\sqrt{M^2 + \Xi^2_{\text{RES}}(\sqrt{M^2 + \Xi^2_{\text{RES}}} + M)}} = \frac{\Xi^2_{\text{RES}}\Lambda_{\text{RES}}}{2A_i\sqrt{M^2 + \Xi^2_{\text{RES}}(\sqrt{M^2 + \Xi^2_{\text{RES}}} + M)}} > 0. \]

Hence, \( \Lambda_{\text{RES}} \) is increasing in \( K, A_1 \) and \( A_2 \) but decreasing in \( \theta_1 \) and \( \theta_2 \).
Second, we derive the comparative statistics of $K_{i}^{\text{RES}}$. Without loss of generality, we let $i = 1$. Then,

\[
\frac{\partial K_{1}^{\text{RES}}}{\partial K} = \frac{\sqrt{A_{1} \theta_{1}}}{\sqrt{A_{1} \theta_{1} + \sqrt{A_{2} \theta_{2}}}} > 0,
\]

\[
\frac{\partial K_{1}^{\text{RES}}}{\partial A_{1}} = \sqrt{A_{2} \theta_{2}} - \frac{\theta_{1} \sqrt{A_{2} \theta_{2}}}{2 \sqrt{A_{1} \theta_{1}} (\sqrt{A_{1} \theta_{1} + \sqrt{A_{2} \theta_{2}}})^2} (A_{1} + A_{2} - K) > 0,
\]

\[
\frac{\partial K_{1}^{\text{RES}}}{\partial A_{2}} = -\frac{\theta_{2} \sqrt{A_{1} \theta_{1}}}{2 \sqrt{A_{2} \theta_{2}} (\sqrt{A_{1} \theta_{1} + \sqrt{A_{2} \theta_{2}}})^2} (A_{1} + A_{2} - K) < 0.
\]

Hence, $K_{i}^{\text{RES}}$ is increasing in $K$ and $A_{i}$ but decreasing in $A_{-i}$. Since $K_{i}^{\text{RES}}$ is increasing in $K$, by Proposition 1, $\lambda_{i}^{\text{RES}}$ is increasing in $K$. Since $K_{-i}^{\text{RES}}$ is decreasing in $A_{i}$, by Proposition 1, $\lambda_{-i}^{\text{RES}}$ is decreasing in $A_{i}$. However, since $\Lambda^{\text{RES}} = \lambda_{i}^{\text{RES}} + \lambda_{-i}^{\text{RES}}$ is increasing in $A_{i}$, $\lambda_{i}^{\text{RES}}$ must be increasing in $A_{i}$. Notice that $\mu_{i}^{\text{RES}} = \frac{1}{2} (K_{i}^{\text{RES}} + \lambda_{i}^{\text{RES}})$. Then $\mu_{i}^{\text{RES}}$ is also increasing in $K$ and $A_{i}$ but decreasing in $A_{-i}$.

For the comparative statics with respect to $\theta_{1}$ and $\theta_{2}$, we have

\[
\frac{\partial K_{1}^{\text{RES}}}{\partial \theta_{1}} = -\frac{A_{1} \sqrt{A_{2} \theta_{2}}}{2 \sqrt{A_{1} \theta_{1}} (\sqrt{A_{1} \theta_{1} + \sqrt{A_{2} \theta_{2}}})^2} (A_{1} + A_{2} - K) < (>) 0, \text{ if } A_{1} + A_{2} > (<) K,
\]

\[
\frac{\partial K_{1}^{\text{RES}}}{\partial \theta_{2}} = \frac{A_{2} \sqrt{A_{1} \theta_{1}}}{2 \sqrt{A_{2} \theta_{2}} (\sqrt{A_{1} \theta_{1} + \sqrt{A_{2} \theta_{2}}})^2} (A_{1} + A_{2} - K) > (<) 0, \text{ if } A_{1} + A_{2} > (<) K.
\]

Hence, $K_{i}^{\text{RES}}$ is increasing (decreasing) in $\theta_{-i}$ but decreasing (increasing) in $\theta_{i}$, if $A_{1} + A_{2} > (<) K$.

If $A_{1} + A_{2} > K$, since $K_{i}^{\text{RES}}$ is increasing in $\theta_{i}$, by Proposition 1, $\lambda_{i}^{\text{RES}}$ is increasing in $\theta_{i}$. However, since $\Lambda^{\text{RES}}$ is decreasing in $\theta_{i}$, $\lambda_{i}^{\text{RES}}$ must be decreasing in $\theta_{i}$. Hence, $\lambda_{i}^{\text{RES}}$ is increasing in $\theta_{-i}$ but decreasing in $\theta_{i}$. Notice that $\mu_{i}^{\text{RES}} = \frac{1}{2} (K_{i}^{\text{RES}} + \lambda_{i}^{\text{RES}})$. Then $\mu_{i}^{\text{RES}}$ is also increasing in $\theta_{-i}$ but decreasing in $\theta_{i}$.

If $A_{1} + A_{2} < K$, since $K_{i}^{\text{RES}}$ is decreasing in $\theta_{i}$, by Proposition 1, $\lambda_{i}^{\text{RES}}$ is decreasing in $\theta_{i}$. Hence, $\mu_{i}^{\text{RES}}$ is also decreasing in $\theta_{i}$ since $\Theta^{\text{RES}}$ is increasing in $\theta_{i}$, by Theorem 2, $\lambda_{i}^{\text{RES}}$ and $\mu_{i}^{\text{RES}}$ are decreasing in $\theta_{i}$. \(\blacksquare\)

**Proof of Theorem 3.** For the reservation strategy, the facility provider maximizes $\lambda_{i}(K_{1}, \mu_{i}) + \lambda_{2}(K_{2}, \mu_{2})$ subject to $K_{1} + K_{2} = K$, where $\lambda_{i}(K_{i}, \mu_{i})$ is defined in Equation (2). For the pooling strategy, the facility provider maximizes $\lambda_{i}(K - \mu_{2}, \mu_{i}) + \lambda_{2}(K - \mu_{1}, \mu_{2})$. Since $\lambda_{i}(K_{i}, \mu_{i})$ is increasing in $K_{i}$ and $K_{1} + K_{2} = K$, $\lambda_{i}(K_{i}, \mu_{i}) < \lambda_{i}(K - \mu_{-i}, \mu_{i})$, where $\mu_{i} < K_{i}$ and $i = 1$ or 2. This implies the claim. \(\blacksquare\)
**Proof of Theorem 4.** By Theorem 2, it is optimal for the facility provider to serve both service providers than to leave out one service provider if Assumptions 1 and 2 hold. By Theorem 3, the facility provider should do better by using the pooling strategy than by using the reservation strategy under the centralized system. Hence, it is optimal for the facility provider to serve both service providers by using the pooling strategy, that is, $\lambda_i(K - \mu_{i+}^\text{CEN}, \mu_i^\text{CEN}^*) > 0$ for $i = 1, 2$.

By Equation (2), we have

$$\Lambda^\text{CEN}(\mu_1, \mu_2) = \frac{1}{2} \left( A_1 + A_2 - \frac{A_1 \theta_1 + A_2 \theta_2}{K - \mu_1 - \mu_2} + \mu_1 + \mu_2 \right) - \frac{1}{2} \sum_{i=1}^{2} \sqrt{\delta_i^2 + 4A_i \theta_i}$$

$$= \frac{1}{2} \left[ A_1 + A_2 + w - \frac{\beta_1 + \beta_2}{K - w} \right] - \frac{1}{2} \sum_{i=1}^{2} \sqrt{\delta_i^2 + 4\beta_i},$$

where $\delta_i = A_i - \frac{A_i \theta_i}{K - \mu_i - \mu_2} - \mu_i$, $\beta_i = A_i \theta_i$ and $w = \mu_1 + \mu_2$.

First, we fix $w$ and maximize $\Lambda^\text{CEN}(\mu_1, \mu_2)$. This is equivalent to minimizing $\sum_{i=1}^{2} \sqrt{\delta_i^2 + 4\beta_i}$ subject to $\mu_1 + \mu_2 = w$. Notice that the objective function is convex in $(\mu_1, \mu_2)$ given the constraint. We let $z$ be the Lagrange multiplier and construct the Lagrange function

$$f(\mu_1, \mu_2, z) = \sum_{i=1}^{2} \sqrt{\delta_i^2 + 4\beta_i} + z(\mu_1 + \mu_2 - w).$$

Notice that $\frac{\partial \delta_i}{\partial \mu_i} = -1$ when $w$ is fixed. Letting $\frac{\partial f(\mu_1, \mu_2, z)}{\partial \mu_i} = 0$, we have $\frac{\delta_i}{\sqrt{\delta_i^2 + 4\beta_i}} = z$. This implies that $\delta_i = \frac{2z}{\sqrt{1-z^2}} \sqrt{\beta_i}$ for $i = 1, 2$, and $\sum_{i=1}^{2} \sqrt{\delta_i^2 + 4\beta_i} = \frac{2}{\sqrt{1-z^2}} (\sqrt{\beta_1} + \sqrt{\beta_2})$.

Since $\delta_i = A_i - \frac{\beta_i}{K - w} - \mu_i$, $\beta_i = A_i \theta_i - \delta_i$. Since $\mu_1 + \mu_2 = w$, we have $\sum_{i=1}^{2} \left( A_i - \frac{\beta_i}{K - w} \right) - w = \frac{2z}{\sqrt{1-z^2}} (\sqrt{\beta_1} + \sqrt{\beta_2})$. Letting

$$\Delta = \frac{A_1 + A_2 - \frac{\beta_1 + \beta_2}{K - w} - w}{2 \left( \sqrt{\beta_1} + \sqrt{\beta_2} \right)},$$

we find that $z = \Delta/\sqrt{1 + \Delta^2}$. Then $\sum_{i=1}^{2} \sqrt{\delta_i^2 + 4\beta_i} = 2 \left( \sqrt{\beta_1} + \sqrt{\beta_2} \right) \sqrt{1 + \Delta^2}$. Hence,

$$\Lambda^\text{CEN} = \frac{1}{2} \left[ A_1 + A_2 + w - \frac{\beta_1 + \beta_2}{K - w} - 2 \left( \sqrt{\beta_1} + \sqrt{\beta_2} \right) \sqrt{1 + \Delta^2} \right].$$

Notice that $\frac{\partial \Delta}{\partial w} = -\frac{1}{2(\sqrt{\beta_1} + \sqrt{\beta_2})} \left[ \frac{\beta_1 + \beta_2}{(K - w)^2} + 1 \right]$. Letting $\frac{\partial \Lambda^\text{CEN}}{\partial w} = 0$, we have

$$\frac{\beta_1 + \beta_2}{(K - w)^2} = \frac{\sqrt{1 + \Delta^2} + \Delta}{\sqrt{1 + \Delta^2} - \Delta} = \left( \frac{\sqrt{1 + \Delta^2} + \Delta}{\sqrt{1 + \Delta^2} - \Delta} \right)^2.$$

Since $\Delta$ is decreasing in $w$ and $\sqrt{1 + \Delta^2} + \Delta$ is positive and increasing in $\Delta$, the right side of the FOC equation is decreasing in $w$. Since the left side of the FOC equation is increasing in $w$ and $\lim_{w \to K} \Lambda^\text{CEN} = -\infty$, $\Lambda^\text{CEN}$ is quasi-concave in $w$. By the FOC equation, we have $\frac{\beta_1 + \beta_2}{(K - w)^2} = \ldots$
\( \sqrt{\beta_1 + \beta_2} \left( \sqrt{1 + \Delta^2 + \Delta} \right) \) and \( w = K - \sqrt{\beta_1 + \beta_2} \left( \sqrt{1 + \Delta^2 - \Delta} \right) \). Plugging them into Equation (13), we have \( \Delta = \frac{A_1 + A_2 - K}{2(\sqrt{\beta_1 + \beta_2} + \sqrt{\beta_1 + \beta_2})} = \frac{M}{\Xi_{\text{CEN}}} \). Plugging them into Equation (14), we have

\[
\Lambda_{\text{CEN}}^\ast = \frac{1}{2} \left[ A_1 + A_2 + K - \Xi_{\text{CEN}} \sqrt{1 + \Delta^2} \right] = \frac{1}{2} \left[ A_1 + A_2 + K - \sqrt{M^2 + \Xi_{\text{CEN}}^2} \right].
\]

Since \( z = \Delta / \sqrt{1 + \Delta^2} \) and \( \frac{\delta_i}{\sqrt{\delta_i^2 + 4\beta_i}} = z \), we have \( \delta_i = 2\sqrt{\beta_i}\Delta \) and \( \delta_i^2 + 4\beta_i = 2\sqrt{\beta_i}(1 + \Delta^2) \). Hence, for both \( i = 1, 2 \),

\[
\mu_i^\ast_{\text{CEN}} = A_i - \frac{\beta_i}{K - w} - \delta_i = A_i - \frac{\beta_i}{\sqrt{\beta_1 + \beta_2}} \sqrt{1 + \Delta^2} - \left( \frac{\beta_i}{
\sqrt{\beta_1 + \beta_2}} + 2\sqrt{\beta_i} \right) \Delta
\]

\[
= A_i - \frac{1}{\Xi_{\text{CEN}}} \left[ \frac{\beta_i}{\sqrt{\beta_1 + \beta_2}} \left( M + \sqrt{M^2 + \Xi_{\text{CEN}}^2} \right) + 2\sqrt{\beta_i} M \right],
\]

\[
\lambda_i^\ast_{\text{CEN}} = \frac{1}{2} \left[ A_i - \frac{\beta_i}{K - w} + \mu_i - \beta_i^2 + 4\beta_i \right] = A_i - \left( \frac{\beta_i}{\sqrt{\beta_1 + \beta_2}} + \sqrt{\beta_i} \right) \left( \sqrt{1 + \Delta^2} + \Delta \right)
\]

\[
A_i - \frac{1}{\Xi_{\text{CEN}}} \left( \frac{\beta_i}{\sqrt{\beta_1 + \beta_2}} + \sqrt{\beta_i} \right) \left( M + \sqrt{M^2 + \Xi_{\text{CEN}}^2} \right).
\]

This concludes the proof of the theorem.

**Proof of Proposition 4.** First, we derive the comparative statics of \( \Lambda_{\text{CEN}}^\ast \). Since \( \Xi_{\text{CEN}} \) is increasing in \( \theta_i, \Lambda_{\text{CEN}}^\ast \) is decreasing in \( \theta_i \). Furthermore, \( \frac{\partial \Lambda_{\text{CEN}}^\ast}{\partial K} = \frac{1}{2} \left( 1 + M/\sqrt{M^2 + \Xi_{\text{CEN}}^2} \right) > 0 \) and

\[
\frac{\partial \Lambda_{\text{CEN}}^\ast}{\partial A_i} = \frac{1}{2} \left( 1 - \frac{M + (1/\sqrt{A_1 \theta_1 + A_2 \theta_2 + 1/\sqrt{A_1 \theta_1}}) \Xi_{\text{CEN}}}{\sqrt{M^2 + \Xi_{\text{CEN}}^2}} \right)
\]

\[
= \Xi_{\text{CEN}}^2 - (1/\sqrt{A_1 \theta_1 + A_2 \theta_2 + 1/\sqrt{A_1 \theta_1}}) \Xi_{\text{CEN}} (\sqrt{M^2 + \Xi_{\text{CEN}}^2} + M)
\]

\[
= \frac{\Xi_{\text{CEN}}^2 - \Xi_{\text{CEN}}^2 \Lambda_{\text{CEN}}^\ast}{2A_i \sqrt{M^2 + \Xi_{\text{CEN}}^2} (\sqrt{M^2 + \Xi_{\text{CEN}}^2} + M)} > 0.
\]

Hence, \( \Lambda_{\text{CEN}}^\ast \) is increasing in \( K, A_1 \) and \( A_2 \) but decreasing in \( \theta_1 \) and \( \theta_2 \).

Second, we derive the comparative statics of \( \lambda_i^\ast_{\text{CEN}} \) and \( \mu_i^\ast_{\text{CEN}} \). Since \( \frac{\partial}{\partial \theta_i} \left( M + \sqrt{M^2 + \Xi_{\text{CEN}}^2} \right) = - \left( 1 + M/\sqrt{M^2 + \Xi_{\text{CEN}}^2} \right) < 0 \), \( \lambda_i^\ast_{\text{CEN}} \) and \( \mu_i^\ast_{\text{CEN}} \) are increasing in \( K \).

**Proof of Theorem 5.** By Theorems 1, 2 and 4, it suffices to compare \( \Xi_{\text{CEN}}, \Xi_{\text{POOL}} \) and \( \Xi_{\text{RES}} \). Notice that \( \Xi_{\text{CEN}}^2 = 4\beta_2 (1 + \sqrt{\gamma_1} + \sqrt{1 + \gamma_1})^2 \), \( \Xi_{\text{POOL}}^2 = 24\beta_2 (1 + \gamma_1) \) and \( \Xi_{\text{RES}}^2 = 16\beta_2 (1 + \sqrt{\gamma_1})^2 \).

First, we prove that \( \Xi_{\text{POOL}} > \Xi_{\text{CEN}} \). Notice that

\[
\Xi_{\text{POOL}}^2 - \Xi_{\text{CEN}}^2 = 4\beta_2 \left[ 6(1 + \gamma_1) - \left( 1 + \sqrt{\gamma_1} + \sqrt{1 + \gamma_1} \right)^2 \right]
\]

\[
= 4\beta_2 \left[ \sqrt{6} \sqrt{1 + \gamma_1} + \left( 1 + \sqrt{\gamma_1} + \sqrt{1 + \gamma_1} \right) \right] \left[ \sqrt{6} \sqrt{1 + \gamma_1} - \left( 1 + \sqrt{\gamma_1} + \sqrt{1 + \gamma_1} \right) \right].
\]

Then it suffices to prove that \( (\sqrt{6} - 1)(1 + \sqrt{\gamma_1}) > 0 \). Notice that

\[
\left( (\sqrt{6} - 1) \sqrt{1 + \gamma_1} \right)^2 - (1 + \sqrt{\gamma_1})^2 = (6 - 2\sqrt{6})(1 + \gamma_1) - 2\sqrt{1 + \gamma_1} > (1 + \gamma_1) - 2\sqrt{1 + \gamma_1} = (1 - \sqrt{\gamma_1})^2 \geq 0.
\]
Hence, $\Xi_{\text{POOL}} > \Xi_{\text{CEN}}$.

Second, we prove that $\Xi_{\text{RES}} > \Xi_{\text{CEN}}$. Notice that

$$\Xi_{\text{RES}}^2 - \Xi_{\text{CEN}}^2 = 4\beta_2 \left[ 4 \left( 1 + \sqrt{\gamma_1} \right)^2 - (1 + \sqrt{\gamma_1} + \sqrt{1 + \gamma_1})^2 \right]$$

$$= 4\beta_2 \left[ 3 \left( 1 + \sqrt{\gamma_1} \right) + \sqrt{1 + \gamma_1} \right] \left( 1 + \sqrt{\gamma_1} - \sqrt{1 + \gamma_1} \right).$$

Then it suffices to prove that $(1 + \sqrt{\gamma_1}) - \sqrt{1 + \gamma_1} > 0$. Notice that $(1 + \sqrt{\gamma_1})^2 - (\sqrt{1 + \gamma_1})^2 = 2\sqrt{\gamma_1} > 0$. Hence, $\Xi_{\text{POOL}} > \Xi_{\text{CEN}}$.

Finally, we compare $\Xi_{\text{POOL}}$ and $\Xi_{\text{RES}}$. Notice that

$$\Xi_{\text{POOL}}^2 - \Xi_{\text{RES}}^2 = 8\beta_2 \left[ 3(1 + \gamma_1) - 2(1 + \sqrt{\gamma_1})^2 \right] = 8\beta_2 \left[ (\sqrt{\gamma_1})^2 - 4\sqrt{\gamma_1} + 1 \right].$$

Hence, $\Xi_{\text{POOL}} < \Xi_{\text{RES}}$ if $2 - \sqrt{3} < \sqrt{\gamma_1} < 2 + \sqrt{3}$ (i.e., $7 - 4\sqrt{3} < \gamma_1 < 7 + 4\sqrt{3}$), $\Xi_{\text{POOL}} = \Xi_{\text{RES}}$ if $\sqrt{\gamma_1} = 2 \pm \sqrt{3}$ (i.e., $\gamma_1 = 7 \pm 4\sqrt{3}$), and $\Xi_{\text{POOL}} > \Xi_{\text{RES}}$ if $\sqrt{\gamma_1} < 2 - \sqrt{3}$ or $\sqrt{\gamma_1} > 2 + \sqrt{3}$ (i.e., $\gamma_1 < 7 - 4\sqrt{3}$ or $\gamma_1 > 7 + 4\sqrt{3}$).

By Theorems 1, 2 and 4, it is clear that the order of $\Lambda_{\text{CEN}}^*$, $\Lambda_{\text{POOL}}^*$ and $\Lambda_{\text{RES}}^*$ is the same as the reverse order of $\Xi_{\text{CEN}}, \Xi_{\text{POOL}}$ and $\Xi_{\text{RES}}$.

**Proof of Theorem 6.** By Theorems 1, 2 and 4, we have

$$\mu_{\text{POOL}}^* = A_1 + A_2 - M - \frac{1}{6} \left( \sqrt{M^2 + \Xi_{\text{POOL}}^2} - M \right),$$

$$\mu_{\text{RES}}^* = A_1 + A_2 - M - \frac{1}{4} \left( \sqrt{M^2 + \Xi_{\text{RES}}^2} - M \right),$$

$$\mu_{\text{CEN}}^* = A_1 + A_2 - M - \frac{\sqrt{\beta_1 + \beta_2}}{\Xi_{\text{CEN}}} \left( \sqrt{M^2 + \Xi_{\text{CEN}}^2} - M \right),$$

where $\beta_i = A_i \theta_i$.

First, we show that $\mu_{\text{POOL}}^* > \mu_{\text{CEN}}^*$. Let $\delta = 6\sqrt{\beta_1 + \beta_2} / \Xi_{\text{CEN}}$. Then $6 \left( \mu_{\text{POOL}}^* - \mu_{\text{CEN}}^* \right) = \delta \left( \sqrt{M^2 + \Xi_{\text{CEN}}^2} - M \right) - \left( \sqrt{M^2 + \Xi_{\text{POOL}}^2} - M \right)$. Note that

$$\delta = 6\sqrt{\beta_1 + \beta_2} / \Xi_{\text{CEN}} = 3\frac{\sqrt{\beta_1 + \beta_2}}{\sqrt{\beta_1 + \beta_2 + \sqrt{\beta_1 + \beta_2}}} > 1,$$

$$3 - 2\delta = 3 - 12\frac{\sqrt{\beta_1 + \beta_2}}{\Xi_{\text{CEN}}} = 3\frac{\sqrt{\beta_1 + \beta_2} - \sqrt{\beta_1 + \beta_2}}{\sqrt{\beta_1 + \beta_2 + \sqrt{\beta_1 + \beta_2}}} > 0.$$

If $M \geq 0$, $6 \left( \mu_{\text{POOL}}^* - \mu_{\text{CEN}}^* \right) = \delta \sqrt{M^2 + \Xi_{\text{CEN}}^2} - \left( \delta - 1 \right)M + \sqrt{M^2 + \Xi_{\text{POOL}}^2}$. Since

$$\delta^2 \left( M^2 + \Xi_{\text{CEN}}^2 \right) - \left( \delta - 1 \right)M + \sqrt{M^2 + \Xi_{\text{POOL}}^2}^2 = 2 \left[ 6(\beta_1 + \beta_2) + (\delta - 1)M^2 - (\delta - 1)M \frac{M^2 + \Xi_{\text{POOL}}^2}{\sqrt{M^2 + \Xi_{\text{POOL}}^2}} \right]$$

and

$$\left[ 6(\beta_1 + \beta_2) + (\delta - 1)M^2 \right] - (\delta - 1)^2 M^2 \frac{M^2 + \Xi_{\text{POOL}}^2}{\sqrt{M^2 + \Xi_{\text{POOL}}^2}} = 36(\beta_1 + \beta_2)^2 + 12(\beta_1 + \beta_2)(\delta - 1)(3 - 2\delta)M^2 > 0,$$

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we have $\mu^{\text{POOL}} > \mu^{\text{CEN}}$. 

If $M < 0$, 6 (\(\mu^{\text{POOL}} - \mu^{\text{CEN}}\)) = \left[ \delta \sqrt{M^2 + \Xi^2_{\text{CEN}}} - (\delta - 1)M \right] - \sqrt{M^2 + \Xi^2_{\text{POOL}}}$. Since

\[
\left[ \delta \sqrt{M^2 + \Xi^2_{\text{CEN}}} - (\delta - 1)M \right]^2 - \left( M^2 + \Xi^2_{\text{POOL}} \right) = 12(\beta_1 + \beta_2) - 2\delta(\delta - 1)M \left( \sqrt{M^2 + \Xi^2_{\text{CEN}}} - M \right) > 0,
\]

we have $\mu^{\text{POOL}} > \mu^{\text{CEN}}$. Hence, $\mu^{\text{POOL}} > \mu^{\text{CEN}}$. 

Second, we show $\mu^{\text{CEN}} > \mu^{\text{RES}}$. It is easy to check that $\frac{4\sqrt{\beta_1 + \beta_2}}{\Xi_{\text{CEN}}^{\ast}} < 1$. Since $\Xi_{\text{RES}} > \Xi_{\text{CEN}}$, then

\[
\sqrt{M^2 + \Xi^2_{\text{RES}}} - M > \sqrt{M^2 + \Xi^2_{\text{CEN}}} - M > \frac{4\sqrt{\beta_1 + \beta_2}}{\Xi_{\text{CEN}}^{\ast}} \left( \sqrt{M^2 + \Xi^2_{\text{CEN}}} - M \right). 
\]

Hence, $\mu^{\text{CEN}} > \mu^{\text{RES}}$. $\square$

**Proof of Theorem 7.** Since the indices of the two service providers are arbitrary, we only prove the conclusions of the theorem for $i = 1$. We first prove $\gamma_1 \in (1, 4)$, which is equivalent to proving that $\lambda_1^{\text{POOL}} > \lambda_1^{\text{RES}}$ if $\gamma_1 \leq 1$, and $\lambda_1^{\text{POOL}} < \lambda_1^{\text{RES}}$ if $\gamma_1 \geq 4$. 

Since $\Xi_{\text{POOL}} = \sqrt{24(\beta_1 + \beta_2)}$ and $\Xi_{\text{RES}} = 4 \left( \sqrt{\beta_1 + \beta_2} \right)$,

\[
\lambda_1^{\text{POOL}} = A_1 - \beta_1 \cdot \frac{M + \sqrt{M^2 + \Xi^2_{\text{POOL}}}}{2(\beta_1 + \beta_2)} \quad (15)
\]

\[
= A_1 - \beta_1 \cdot \frac{12}{\sqrt{M^2 + \Xi^2_{\text{POOL}}} - M}, \quad (16)
\]

\[
\lambda_1^{\text{RES}} = A_1 - \beta_1 \cdot \frac{M + \sqrt{M^2 + \Xi^2_{\text{RES}}}}{2\sqrt{\beta_1} \left( \sqrt{\beta_1 + \beta_2} \right)} \quad (17)
\]

\[
= A_1 - \beta_1 \cdot \frac{8 \left( 1 + \sqrt{1/\gamma_1} \right)}{\sqrt{M^2 + \Xi^2_{\text{RES}}} - M}, \quad (18)
\]

In the proof of Theorem 5, we proved that $\Xi_{\text{POOL}} < \Xi_{\text{RES}}$ if $7 - 4\sqrt{3} < \gamma_1 < 7 + \sqrt{3}$ and $\Xi_{\text{POOL}} \geq \Xi_{\text{RES}}$ otherwise.

First, we consider $\gamma_1 \leq 1$. When $7 - 4\sqrt{3} < \gamma_1 \leq 1$, $\Xi_{\text{POOL}} < \Xi_{\text{RES}}$. Then $M + \sqrt{M^2 + \Xi^2_{\text{POOL}}} < M + \sqrt{M^2 + \Xi^2_{\text{RES}}}$ and $2(\beta_1 + \beta_2) \geq 2\sqrt{\beta_1} \left( \sqrt{\beta_1 + \beta_2} \right)$. By Equations (15) and (17), $\lambda_1^{\text{POOL}} > \lambda_1^{\text{RES}}$. When $\gamma_1 \leq 7 - 4\sqrt{3}$, $\Xi_{\text{POOL}} \geq \Xi_{\text{RES}}$. Then $\sqrt{M^2 + \Xi^2_{\text{POOL}}} - M \geq \sqrt{M^2 + \Xi^2_{\text{RES}}} - M$ and $12 \geq 8 \left( 1 + \sqrt{1/\gamma_1} \right)$. By Equations (16) and (18), $\lambda_1^{\text{POOL}} \geq \lambda_1^{\text{RES}}$. Hence, $\lambda_1^{\text{POOL}} > \lambda_1^{\text{RES}}$ when $\gamma_1 \leq 1$.

Second, we consider $\gamma_1 \geq 4$. When $4 \leq \gamma_1 < 7 + 4\sqrt{3}$, $\Xi_{\text{POOL}} < \Xi_{\text{RES}}$. Then $\sqrt{M^2 + \Xi^2_{\text{POOL}}} - M < \sqrt{M^2 + \Xi^2_{\text{RES}}} - M$ and $12 \geq 8 \left( 1 + \sqrt{1/\gamma_1} \right)$. By Equations (16) and (18), $\lambda_1^{\text{POOL}} < \lambda_1^{\text{RES}}$. When $\gamma_1 \geq 7 + 4\sqrt{3}$, $\Xi_{\text{POOL}} \geq \Xi_{\text{RES}}$. Then $M + \sqrt{M^2 + \Xi^2_{\text{POOL}}} \geq M + \sqrt{M^2 + \Xi^2_{\text{RES}}}$ and $2(\beta_1 + \beta_2) < 2\sqrt{\beta_1} \left( \sqrt{\beta_1 + \beta_2} \right)$. By Equations (15) and (17), $\lambda_1^{\text{POOL}} < \lambda_1^{\text{RES}}$. Hence, $\lambda_1^{\text{POOL}} < \lambda_1^{\text{RES}}$ when $\gamma_1 \geq 4$.

Now we prove the main statement of the theorem. Notice that

\[
\lambda_1^{\text{RES}} - \lambda_1^{\text{POOL}} = \beta_1 \cdot \frac{M + \sqrt{M^2 + \Xi^2_{\text{POOL}}}}{2(\beta_1 + \beta_2)} - \beta_1 \cdot \frac{M + \sqrt{M^2 + \Xi^2_{\text{RES}}}}{2\sqrt{\beta_1} \left( \sqrt{\beta_1 + \beta_2} \right)}
\]

\[
= \frac{\sqrt{\beta_2}}{2} \left\{ \frac{x^2}{x^2 + 1} \left[ \sqrt{N^2 + 24(x^2 + 1) + N} \right] - \frac{x}{x + 1} \left[ \sqrt{N^2 + 16(x + 1)^2 + N} \right] \right\}.
\]
where $N = M/\sqrt{\beta}$ and $x = \sqrt{\gamma}$.

Let $f(x) = \frac{x^2}{x+1} \left[ \sqrt{N^2 + 24(x^2 + 1)} + N \right] - \frac{x}{x+1} \left[ \sqrt{N^2 + 16(x+1)^2} + N \right]$. Since $\lambda^\text{RES*}_1 - \lambda^\text{POOL*}_1 < 0$ when $\gamma_1 \leq 1$ and $\lambda^\text{RES*}_1 - \lambda^\text{POOL*}_1 > 0$ when $\gamma_1 \geq 4$, $f(x) > 0$ when $x \geq 2$ and $f(x) < 0$ when $x \leq 1$. Hence, it suffices to prove that $f'(x) > 0$ when $x \in [1, 2]$.

Let $A = \sqrt{N^2 + 24(x^2 + 1)}$ and $B = \sqrt{N^2 + 16(x+1)^2}$. Then

$$f'(x) = \frac{2x}{(x^2+1)^2}(A+N) + \frac{x^2}{x^2+1} \frac{24x}{A} - \frac{1}{(x+1)^2}(B+N) - \frac{x}{x+1} \frac{16(x+1)}{B}$$

$$= \frac{24x}{(x^2+1)^2} + \frac{24x^3}{A} - \frac{16}{B} - \frac{16x}{B}$$

$$= 16 \left( \frac{1}{A-N} - \frac{1}{B-N} \right) + 16x \left( \frac{1}{A} - \frac{1}{B} \right) + \left( \frac{48x}{x^2+1} - 16 \right) \frac{1}{A-N} + \left( \frac{24x^3}{x^2+1} - 16x \right) \frac{1}{A-N}$$

Since $B > A > 0$ and $B - N > A - N > 0$ when $x \in [1, 2]$, it suffices to prove that

$$g(x) = \left( \frac{48x}{x^2+1} - 16 \right) \frac{1}{A-N} + \left( \frac{24x^3}{x^2+1} - 16x \right) \frac{1}{A} > 0.$$

Notice that $A > |N|$. Then $\frac{1}{A-N} > \frac{1}{2\pi}$. Since $\frac{48x}{x^2+1} - 16 > 0$ when $x \in [1, 2]$,

$$g(x) > \left( \frac{48x}{x^2+1} - 16 \right) \frac{1}{2A} + \left( \frac{24x^3}{x^2+1} - 16x \right) \frac{1}{A} = \frac{8(x-1)}{A} \geq 0.$$

This concludes the proof of the theorem.

References


Chayet, S., W. Hopp. 2007. Sequential entry with capacity, price, and lead time competition. Working paper, Washington University, St. Louis, MO.


